MIT Sloan School of Management

MIT Sloan School Working Paper 5236-17

Peer-to-Peer Markets with Bilateral Ratings

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Last Revised: September 2017

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September 2017

[*]The authors thank Alexei Parakhonyak, Duncan Simester, Wenbo Wang, Juanjuan Zhang, and Tianxin Zou for helpful discussions and suggestions. This paper has also benefited from the comments of seminar participants at Peking University, Hong Kong University of Science and Technology, and attendees of 8th Workshop on Consumer Search and Switching Cost, 3rd Workshop on Marketplace Innovation, 2017 Marketing Science Conference, 2017 Summer Institute in Competitive Strategy, and 2017 Northeast Marketing Conference. We are grateful for financial assistance from the NET Institute www.netinst.org. All remaining errors are our own.

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Abstract

We consider a platform that matches service providers with potential customers. Ratings of a service provider reveal the quality of his service while ratings of a consumer reveal the cost to serve her. Under a competitive search framework, we study how bilateral ratings influence market competition and segmentation. Two types of equilibria exist under bilateral ratings. In the first type, low-cost consumers only apply to high-quality service providers, who post a higher price, have longer queues and are less likely to accept an application than low-quality providers. High-cost consumers apply to all service providers and have a lower acceptance rate. In the second type of equilibria, both high- and low-quality service providers serve all consumers. Across all equilibria, equilibrium prices *may decrease* as the fraction of high-quality providers increases, as consumers become more costly to serve, and as the platform's commission rate increases. Compared with a platform with unilateral ratings where only service providers are rated, a platform with bilateral ratings *may* soften service providers' competition, leading to higher equilibrium prices. Lastly, we find that in the case of incomplete market coverage, high-quality service providers *may* charge lower prices than low-quality providers in equilibrium, because by charging a lower price, a high-quality service provider attracts more consumer applications, which enables him to cherrypick a low-cost consumer, while a low-quality service provider faces with consumers with higher serving costs and thus charge a higher price to make up the serving cost.

Keywords: *Platform; Peer-to-Peer; Competitive Search; Matching; Reviews; Information Disclosure; Segmentation*

1. Introduction

The past two decades have witnessed an explosion of information technology that has given birth to larger, faster, and more geographically diverse marketplaces. Peer-to-peer product sharing or service platforms, in particular, help match consumers with service providers all over the globe. They have emerged for a wide range of products and services, ranging from the old fashioned online marketplaces of used goods (eBay, Craiglist) to the emerging specialized platforms in finance (consumer loans: Prosper, Lending Club; start-up financing: Kickstarter, AngelList; currency exchange: Transferwise, CurrencyFair), transportation (bike: Spinlister; boat: Boatbound, GetMyBoat; rides and cars: Uber, Lyft, BlaBla Car, Turo, Getaround, Zimride, ZemCar), accommodation (short-term rental: Airbnb, Roomorama; working space: Citizen Space; gardens: Shared Earth, Landshare), to specialized labor supply (crafts: Etsy; creative work: Patreon; programming: UpWork; household help: TaskRabbit).

Unlike traditional review systems such as Yelp, on which consumers can write reviews about businesses or service providers, peer-to-peer platforms typically allow both parties of a transaction to leave a review after the transaction. Typically, such reviews include both a rating (e.g., a score or the number of stars) and some detailed verbal descriptions. The totality of the reviews can help resolve some uncertainties about the service providers or the consumers. In this paper, we will use the terms "reviews" and "ratings" interchangeably. A consumer can often choose whether to initiate a transaction based on reviews of the service provider, and a service provider can choose whether to accept a transaction based on reviews of the consumer. For example, on Airbnb, a traveller can decide whether to initiate an application to stay with a particular host, after seeing the profile of his property and customer reviews of his past transactions. A host, upon seeing an application, can decide whether to accept it based on other hosts' reviews of the traveller's previous stays. While reviews of the service provider often contain important information on the unknown quality of the service or product, reviews of a customer can help reveal the cost to serve that customer. One Airbnb host's review of a particular customer, for example, states that:[1] "she overwhelmed them (the hosts) and insisted on free breakfast, having them take the trash out, and run her some errands. She harassed my hosts in person and also calling/texting throughout all hours...Still she complained to Airbnb and got her bill cut in half. Do not host this girl."

When a traveller has such reviews from previous hosts, it indicates to the host that the traveller is of high maintenance and costly to serve. If the host believes that he can easily find better guests, it is only rational for him to decline applications from such costly consumers. Recent

[1]See `http://socawlege.com/7-worst-airbnb-reviews-on-the-internet/3/`, accessed in August 2017.

empirical studies (e.g., Horton 2014, Fradkin 2015) have systematically documented that popular service providers on oDesk and Airbnb get contacted by multiple buyers and often have to reject certain guests due to capacity constraints. The bilateral rating system has become a useful tool that enables providers to segment the market by potentially rejecting certain customers based on their estimated cost to serve. With Uber, a driver has the option to automatically block service requests from riders who have poor reviews or fail other criterion that the drivers can define. On retail platforms such as eBay, the service providers can also determine which buyers to serve and some sellers specifically disallow bidders or buyers with poor reviews. In addition to organic reviews on peer-to-peer platforms, certain service providers can use third-party review sites such as ratemycustomers.com, badbuyerlist.org, and customers2avoid.com to avoid costly customers.

As the success of a peer-to-peer platform critically depends on the credibility of the rating system, most platforms adopt careful measures to make reviews reliable and trustworthy: they typically only allow actual parties involved in transactions to write reviews, and to prevent retaliation (e.g., Nosko and Tadelis 2015, Bolton et al. 2013, Horton and Golden 2015), some platforms do not make the reviews available until both parties have submitted a review to the platform while others offer additional incentives for truthful reporting (Fradkin et al., 2015).[2]

One key advantage of online peer-to-peer marketplace, when compared with traditional service markets with lengthy upfront licensing and screening, is the continuous monitoring that grows organically out of the transaction-based reviews within the system. In fact, a central feature of peer-to-peer platforms is their reliance on user data to match buyers and sellers and to monitor behavior. As the bilateral rating system reduces information asymmetries, reputation is built, and trust-based services become possible while they would not have been in traditional offline markets given the prohibitive search costs. In this paper, we examine how the bilateral ratings change key equilibrium outcomes, particularly pricing and competition, in peer-to-peer markets. Compared with unilateral ratings, we view bilateral ratings as the disclosure of more information of one side of the market to the other side. We are interested in understanding the following research questions. Compared to traditional marketplaces with only unilateral ratings of the sellers or service providers, how does the availability of bilateral ratings affect consumers' incentive to request service and the competition between service providers? How do service providers' equilibrium prices and acceptance rates depend on the rating mechanism? Do service providers benefit from ex ante knowing consumers' serving cost and the ability to reject high-cost consumers? If so, who benefits the most?

[2] `https://www.forbes.com/sites/sethporges/2014/10/17/the-strange-game-theory-of-airbnb-reviews`, accessed in August 2017.

We address these questions by analyzing a matching model with heterogenous service providers and consumers. Providers differ in the quality of their services, which consumers can learn from provider's site description, ratings, etc. Consumers differ in their estimated serving cost, which providers can learn from customer reviews and ratings. In essence, our model zooms into three key micro-aspects of peer-to-peer markets that are often discussed but rarely analyzed in the current literature (e.g., Einav et al. 2016): search frictions, product differentiation, and consumer heterogeneity.

Our model features large numbers of service providers and consumers, with a fixed ratio between the two. Each consumer submits one application to a chosen provider, and each provider has the capacity to serve one consumer. We consider a one-shot game that is rooted in the reality of existing peer-to-peer platforms, in which consumers simultaneously submit their applications and service providers then simultaneously choose which application to accept.

When consumers make their application decisions, they consider the expected utility upon acceptance of the application and the acceptance likelihood. In particular, all consumers would find the same kind of providers desirable, the kind who offers higher net surplus. However, the low-cost consumers are more confident that their applications will be accepted given their good ratings, and are more likely to apply to the desirable providers than the high-cost consumers.

Market inefficiency comes from two sources. First, multiple consumers may apply for the same service provider and all but one get rejected, while some service providers may receive no application. As a result, some agents on both sides of the market get unmatched. This is the so-called *coordination frictions*. An increase in the matching rates enhances total welfare. Second, if the cost to serve a consumer depends on the quality of the service being provided (e.g., an hour of time that is needed to serve a costly consumer may be more costly to the owner of a high-end apartment than that of an inexpensive apartment), search friction may also come from bad matches, matches between good providers and bad consumers. In this case, positive assortment in which low-maintenance consumers are matched with high-quality service providers will enhance social welfare. In our model, we allow the cost to serve a consumer to either depend or not depend on the quality of the service being provided. Therefore, positive sorting may or may not necessarily exist in our setting. Coordination frictions will play a major role in creating market inefficiencies in our model.

Service providers set prices to maximize profit. We show that there exist two kinds of price equilibria under bilateral ratings. In one kind of equilibria, prices are set in a way that high-quality service providers are more attractive in terms of providing higher net surplus to accepted consumers. As a result, low-cost consumers apply only to the high-quality service providers while high-cost consumers apply to all providers. The high-quality providers, as a result, have more attractive applicant pools, longer queues, and a lower acceptance rates than low-quality providers. Interestingly, high-cost consumers may get severely penalized for their bad reputations by a disproportionately low acceptance rate: a high-cost consumer would rather reimburse the service provider of her serving cost in order to be treated as a low-cost consumer. In the second kind of equilibria, both high- and low-quality providers are equally attractive. As a result, both high- and low-cost consumers apply to both of them. Nevertheless, low-cost consumers always have a higher acceptance rate than high-cost consumers.

While the multiplicity of equilibria prevents us from a close-form characterization of comparative statics, numerical analysis reveals interesting patterns. First, equilibrium prices tend to decrease with the ratio of service providers to consumers, the fraction of high-quality service providers, consumers' serving costs, and the platform's commission rate. To understand the comparative statics, it is worth noting an interesting feature of our model—the composition of consumers faced by a service providers is endogenous, which depends on the service provider's price as well as others' prices. When a service provider lowers his price, he becomes more competitive among consumers. Given increased competition, consumers with lower costs are more confident to be selected. As a result, more low-cost consumers will apply to the service provider while less high-cost consumers will stay. Therefore, as a service provider lowers his price, he faces a better consumer pool and a lower average serving cost.

We also analyze the market equilibrium under unilateral ratings where consumers' cost information remains private information. When compared with unilateral ratings, bilateral ratings can raise market prices through enabling cost-based customer segmentation and alleviating competition among service providers. The low-cost consumer derive higher surplus under bilateral ratings because they are more likely to be accepted, and the high-cost consumers derive lower surplus given the higher prices. The overall consumer surplus may go down when customer ratings become available, although total surplus in this case can still increase due to higher prices and provider profits. In general, the welfare implications of bilateral ratings are not a clear-cut—it can increase or decrease total and consumer welfare compared with the case of unilateral ratings.

Finally, we also consider the possibility of incomplete market coverage, where some consumers' serving cost is so high that it is not profitable for certain service providers to serve them. In this case, it is desirable for service providers to avoid these customers. Interestingly, we find that when there is a high fraction of high-quality providers, their prices may turn out to be lower than that of low-quality providers in equilibrium. The intuition is that high-quality service providers charge a low price and attract many consumer applications, so that they can cherrypick a low-cost consumer.

Consequently, low-quality providers get the applications from the most expensive consumers and have to charge a high price to make up the high serving costs.

1.1. Literature Review

Our paper builds on the competitive search literature in labor economics pioneered by Peters (1991), Montgomery (1991), and Burdett et al. (2001). In this framework, firms post jobs and wages, and workers conduct directed search over jobs based on the wages. As its counterpart in our setting, service providers post listings and prices, and consumers conduct directed search over listings based on the prices. Compared with other search and matching models (see Rogerson et al. (2005) for a survey), this framework is suitable for our problem because it unlocks the blackbox of matching functions by explicitly modeling the matching process of heterogeneous agents on both sides of the market. Early literature (e.g., Burdett et al. 2001) focus on homogenous agents on both sides of the market, while more recent papers start incorporating heterogeneity of agents, with a particular attention to the condition for positive assortative matching (e.g., Eeckhout and Kircher 2010, Chade et al. 2017). The most related paper in this literature is Shi (2002), who considers matches between workers with heterogenous skills and firms with heterogenous technologies and assumes that high-skill workers generate higher output when they are matched with high-technology firms. Therefore, there exists a possible positive sorting in Shi (2002), when high-skill workers are matched with high-technology firms. Similar with Shi (2002), we also consider matching between heterogenous agents on both sides of the market, but our market structure is different in that high-cost consumers can be equally costly for both high- and low-quality service providers.[3] In contrast, positive assortment is not the driving force underlying our model. Also, from a modeling perspective, we have a continuous distribution of customer types, which enables a natural characterization of market segmentation, and requires quite different equilibrium analysis techniques.

Our research also contributes to the growing literature on behavior-based price discrimination. In our setting, service providers can learn about their costs to serve a consumer based on either the consumer's past interaction with them or her reviews on the online peer-to-peer platform. This contrasts with the bulk of the literature on behavior-based price discrimination, where firms discriminate based on their customers' willingness to pay that they learn from their past purchases. For example, a firm may discriminate between its own and the competitors' customers (e.g., Pazgal and Soberman 2008, Shin and Sudhir 2010, Zhang 2011). A monopolist firm that is able to recognize

[3]We also allow a consumer to be more costly for high-quality service providers than for low-quality service providers. Our main result of the paper does not depend on the relationship between consumers' costs and service providers' quality.

past customers may also optimally charge returning and new customers different prices since their differing willingness to pay is revealed by past purchases (e.g., Hart and Tirole 1988, Schmidt 1993, Villas-Boas 2004). In our model, firms cannot charge different prices based on the consumers' willingness to pay, nor can the firms tailor different prices based on the consumers' serving cost; however, the firm can decide whether to serve a specific consumer (at the uniform price) based on the cost to serve that consumer. In this respect, our paper relates most closely to Shin et al. (2012), who show that, after learning the customers' costs to serve, a monopolist may find it optimal to fire some high-cost customers even if they are profitable.

Finally, our paper makes a key contribution to the emerging literature on peer-to-peer platforms, collaborative consumption and sharing economy (e.g., Einav et al. 2016, Veiga and Weyl 2017). Existing research in this literature has generally focused on how peer-to-peer markets differ from traditional markets with professional sellers, discussing the impact of peer-to-peer markets on traditional markets (e.g., Zervas and Byers 2016, Jiang and Tian 2016, Tian and Jiang 2017, Gong et al. 2017), value of flexible work (e.g., Chen et al. 2017), impact of choice sets (e.g., Halaburda et al. 2016), search frictions (e.g., Horton 2014, Fradkin 2015, Arnosti et al. 2015), potential incentive misalignment between consumers and the platform (e.g., Armstrong and Zhou 2011, Eliaz and Spiegler 2011, Hagiu and Jullien 2011, De Cornière and Taylor 2014), and comparison of different pricing formats such as auctions, posted prices and surge pricing (e.g., Einav et al. 2015, Gomez Lemmen Meyer 2015, Cullen and Farronato 2015, Guda and Subramanian 2017, Castillo et al. 2017). In terms of the reputation and trust in these markets, the literature has focused on the sellers' incentives to truthfully reveal information (e.g., Jin and Leslie 2003, Jin and Kato 2007, Lewis 2011), the buyers' incentives to leave feedbacks (e.g., Bolton et al. 2013, Nosko and Tadelis 2015, Horton and Golden 2015), and fake reviews (e.g., Mayzlin et al. 2014, Luca and Zervas 2016). While many potential factors may hinder truthful reporting, the flourishing of peer-to-peer markets itself is strong evidence that the reputation systems work well enough to screen out most of the really bad actors and deter highly fraudulent behaviors (e.g., Resnick et al. 2002, Dellarocas 2003, Cabral and Hortacsu 2010). A contemporaneous paper by Romanyuk (2016) is the most related paper, which also investigates the platform's information disclosure problem on peer-to-peer markets. However, his paper is not based on competitive search framework and is quite different from ours. For example, the sellers in his framework do not set prices, which are assumed to be exogenously given. To our best knowledge, our paper is the first theoretical attempt to understand how bilateral ratings in peer-to-peer markets reduce information asymmetry and consequently affect the matching outcomes, equilibrium prices and welfare.

The remainder of the paper proceeds as follows. Section 2 introduces our competitive search model of a peer-to-peer market. Section 3 characterizes the equilibrium outcomes. In Section 4, we compare these outcomes to those in the traditional market, with unilateral ratings, in order to understand the impact of bilateral ratings. Section 5 offers a discussion on robustness of the results by discussing the possibility of incomplete market coverage, which also generates some new insights. Lastly, Section 6 concludes by offering managerial implications of our results and directions for future work.

2. The Model

Consider a market with M service providers distinguished by service quality. A fraction γ of them are of H type and provide a service with high quality q_H; the remaining $1-\gamma$ fraction are of L type and provide a service with low quality q_L, where $1 > \gamma > 0$ and $q_H \geq q_L > 0$. It is assumed that the quality of each provider's service is exogenously given. The fixed cost of service provision for all providers is normalized to zero. Service providers have capacity constraints. Particularly, we assume that each service provider can only serve at most one consumer. An example is hosts on AirBnB, most of whom only have one apartment for lease.

There are N consumers, who differ in their costs to serve. We allow, but do not assume, a consumer's cost to serve to differ for different service providers. Particularly, it is assumed that to serve a consumer of cost type θ, a service provider of quality q incurs cost $\theta g(q)$, where $g(\cdot) > 0$ and $g'(\cdot) \geq 0$. Consumers with a higher θ are more costly to serve. Two special cases are of particular interest. First, $g(q) = 1$, in which case, different service providers incur the same cost to serve a consumer. Second, $g(q) = q$, in which case, a high-quality service provider incurs higher cost than a low-quality service provider when serving the same consumer. We consider below a general setup of $g(\cdot)$ that can incorporate both cases. Consumers' cost type θ follows a distribution function $F(\cdot)$ in $[0, \bar{\theta}]$. The PDF $f(\theta) \equiv F'(\theta)$ exists and it is positive and finite for $\forall \theta \in [0, \bar{\theta}]$. We also assume that consumers are homogeneous in their preferences for service quality. Particularly, a consumer's utility from a service of quality q and price p is given by $u(p, q) = q - p$.

All service providers and consumers meet and trade with one another on the peer-to-peer platform. As with traditional markets, the platform can provide consumers with information on a provider's service quality such as service specifications, descriptions of the service or product in texts, pictures and videos, as well as customer reviews. We thus model providers' service qualities as common knowledge to focus on information disclosure of customer costs. The platform implements a *bilateral review* system, if it also enables service providers to give reviews to consumers and discloses these reviews along with other consumer information to all service providers. We assume that this information fully reveal consumers' cost information to service providers. Therefore, both

q and θ are common knowledge under bilateral ratings. In contrast, a *unilateral review* system is implemented if the platform does not enable service providers to rate consumers or does not disclose consumer information to service providers, in which case, the consumers' cost type θ remains to be private information. We focus on the bilateral review system in the main model and consider the alternative unilateral review system in Section 4.

We consider a matching game in three stages. First, all service providers post prices simultaneously. The posted prices are observable by all agents in the marketplace. It is assumed that service providers cannot price discriminate consumers based on their reviews, i.e., the posted prices do not depend on consumers' cost type θ. This is a reasonable assumption considering most real-world peer-to-peer markets. Following the vast literature of search theory (Rogerson et al. 2005), we assume that service providers can ex ante commit to their posted prices, which also applies to most peer-to-peer markets. Second, all consumers simultaneously submit their applications to service providers. Each consumer can only submit at most one application but is allowed to use a mixed strategy by randomizing the submission of her application across multiple service providers. This means that in equilibrium the consumer applies to any given service provider with an optimized probability. An immediate example one can think of is Airbnb, where consumers can submit their applications to service providers. A consumer almost never submits more than one application for a particular trip as she would have to pay for the stay if the application gets accepted. Lastly, a service provider decides which application to accept if he receives one or more applications from consumers. He accepts at most one application, and when he does accept one, he accepts the consumer with the best reviews, i.e., the lowest θ, to minimize serving cost, because the price has been given and consumers are homogeneous along all other dimensions. Upon his acceptance, the service provider provides service to the consumer and receives the posted price. When a service provider receives no applications or turns down all the applications he receives, he does not get involved in a transaction and receives zero payoff. When a consumer does not submit an application or has her application rejected, she also gets zero payoff. We also assume that the matching platform charges δ percent of the transaction price from each match as a commission fee.

Before proceeding to analyze the game, it is worth noting that in our competitive search framework above, market frictions and mismatches do not result from agents' search costs, instead they originate from coordination frictions: more than one consumer may apply to the same service provider, and some service providers may receive no applications. If we allow unmatched agents to play the same matching game again, some of them would get matched. In theory, we can extend the current one-shot matching game to multiple rounds, which will reduce coordination frictions and mismatches. However, this is not something we intend to model in this paper because coordination frictions are important in many peer-to-peer markets and play a key role in the informational effects of bilateral ratings. Also, competitive search with heterogeneous agents in multiple periods is very difficult to analyze.

To analyze the matching game, we follow the literature (e.g., Butters 1977, Montgomery 1991, Peters 1991, Shi 2002, Shimer 2005) and assume that the market is very large, i.e., $M, N \to \infty$, and neither side of the market is infinitely larger than the other side, i.e., $0 < n \equiv N/M < \infty$. This assumption facilitates the tractability of the model.[4] Given the large market, it is natural to restrict ourselves to symmetric strategies on both sides of the market. That is, we consider the case that all service providers of type $j \in \{H, L\}$ post the same price p_j, and all consumers of type $\theta \in [0, \bar{\theta}]$ use the same application strategy $a_j(\theta)$ ($j \in \{H, L\}$), which is the probability of submitting an application to one particular service provider of type j.

We will solve the game below by backward induction.

2.1. Consumers' Problem

Let us first analyze the consumers' application strategies, given the service providers' qualities q_H and q_L, and the posted prices p_H and p_L. Following the competitive matching literature (e.g., Shi 2002), we define the *queue length* of a service provider of type j as $x_j(\theta) \equiv Nf(\theta)a_j(\theta)$. Subsequently, $x_j(\theta)d\theta$ is the number of applications that the provider j receives from consumers in $[\theta, \theta + d\theta]$. As M and N go to infinity, $a_j(\theta)$ goes to zero while $x_j(\theta)$ converges to a finite positive number. Therefore, it is easier to work with $x_j(\theta)$ than $a_j(\theta)$. Hereafter we work with $x_j(\theta)$ ($j \in \{H, L\}$) and call it consumer θ's application strategy. Intuitively, under a large market, if a subset of consumers with zero measure in $\left[0, \bar{\theta}\right]$ change their application strategies $x_j(\theta)$ ($j \in \{H, L\}$), the market equilibrium will remain unchanged. Therefore, it is reasonable and technically convenient to stipulate that $x_H(\theta)$ and $x_L(\theta)$ are piecewise continuous with a finite number of discontinuities. Each consumer submits one application, so we have the normalization condition $\gamma M a_H(\theta) + (1 - \gamma) M a_L(\theta) = 1$, or equivalently,

$$\gamma x_H(\theta) + (1 - \gamma) x_L(\theta) = nf(\theta). \tag{1}$$

Service providers always prefer consumers with lower serving cost, because their prices have been posted and thus are given. Consider an application from a consumer of type θ to a service provider of type j, this application gets accepted by the service provider if and only if he has not

[4]An equivalent representation of the model is to consider a set of service providers of measure 1 and a set of consumers of measure n. Each individual service provider or consumer is then interpreted as an infinitesimally small subset.

received any application from consumers of types lower than θ. This happens with the following probability

$$b_j(\theta) = \lim_{N\to\infty} \prod_{t=0}^{\theta} (1 - a_j(t))^{Nf(t)\mathrm{d}t} = \lim_{N\to\infty} \prod_{t=0}^{\theta} \left(1 - \frac{x_j(t)}{Nf(t)}\right)^{Nf(t)\mathrm{d}t} = \prod_{t=0}^{\theta} e^{-x_j(t)\mathrm{d}t} = e^{-\int_0^{\theta} x_j(t)\mathrm{d}t}. \quad (2)$$

Obviously, $b_j(\theta)$ decreases with θ, which implies that more costly consumers expect a lower acceptance rate.

A consumer maximizes her expected utility by deciding which service provider to apply to. Let $U(\theta)$ be the maximum expected utility of consumer θ , which is taken as given by individual agents when $N, M \to \infty$. A consumer of type θ submits an offer to a service provider of type j with positive probability if and only if her expected utility from that service provider, $b_j(\theta)(q_j - p_j)$, is equal to or greater than $U(\theta)$. However, it can never be the case that $b_j(\theta)(q_j - p_j) > U(\theta)$. Otherwise, all consumers with cost type θ will submit an offer to that service provider with probability 1, which drives down the acceptance probability $b_j(\theta)$ until $b_j(\theta)(q_j - p_j) = U(\theta)$. Therefore, consumer θ's strategy will be,

$$x_j(\theta) = \begin{cases} \in (0, \infty), & \text{if } b_j(\theta)(q_j - p_j) = U(\theta), \\ 0, & \text{if } b_j(\theta)(q_j - p_j) < U(\theta). \end{cases} \quad (3)$$

Equation (3) illustrates a consumer's tradeoff when deciding which service provider to apply to. On one hand, she prefers service providers that provide a higher utility upon acceptance, $q_j - p_j$; on the other hand, given that all other consumers have the same preference, service providers with higher surplus $q_j - p_j$ are also more competitive and thus may provide a lower acceptance rate $b_j(\theta)$. According to equation (2), we already know that $b_j(\theta)$ decreases with θ. Intuitively, consumers with lower θ are less concerned with the probability of being rejected, because they get rejected only when there exists at least one other consumer with even lower θ who applies to the same service provider. As a result, consumers with low θ will apply to service providers with higher $q_j - p_j$, which drives down the acceptance rate of those service providers. Consequently, consumers with high θ may become indifferent between the two types of service providers, because service providers with higher $q_j - p_j$ deliver a lower acceptance rate while service providers with lower $q_j - p_j$ deliver a higher acceptance rate. Formally, we prove the following theorem, which characterizes consumers' application strategies given the service providers' qualities and posted prices. The proof is in Appendix.

THEOREM 1: *Suppose $q_j - p_j > 0$ for $j \in \{H, L\}$. Define*

$$\theta_H \equiv F^{-1}\left[\frac{\gamma}{n} \ln\left(\frac{q_H - p_H}{q_L - p_L}\right)\right] \text{ and } \theta_L \equiv F^{-1}\left[\frac{1-\gamma}{n} \ln\left(\frac{q_L - p_L}{q_H - p_H}\right)\right].$$

1. *If* $\frac{q_H-p_H}{q_L-p_L} \le e^{-\frac{n}{1-\gamma}}$, *all consumers apply to service providers of type L only. For* $\theta \in \left[0,\bar{\theta}\right]$,

$$x_H(\theta)=0, \quad x_L(\theta)=\frac{n}{1-\gamma}f(\theta),$$
$$U(\theta)=(q_L-p_L)e^{-\frac{n}{1-\gamma}F(\theta)}.$$

2. *If* $e^{-\frac{n}{1-\gamma}} < \frac{q_H-p_H}{q_L-p_L} < 1$, *consumers with type* $\theta \in [0,\theta_L]$ *apply to service providers of type L only, and consumers with* $\theta \in \left(\theta_L,\bar{\theta}\right]$ *apply to both types of service providers. For* $\theta \in \left[0,\bar{\theta}\right]$,

$$x_H(\theta)=\begin{cases} 0, & 0\le\theta\le\theta_L \\ nf(\theta), & \theta_L<\theta\le\bar{\theta} \end{cases}, \quad x_L(\theta)=\begin{cases} \frac{n}{1-\gamma}f(\theta), & 0\le\theta\le\theta_L \\ nf(\theta), & \theta_L<\theta\le\bar{\theta} \end{cases},$$
$$U(\theta)=\begin{cases} (q_L-p_L)e^{-\frac{n}{1-\gamma}F(\theta)}, & 0\le\theta\le\theta_L \\ (q_L-p_L)\left(\frac{q_H-p_H}{q_L-p_L}\right)^{\gamma}e^{-nF(\theta)}, & \theta_L<\theta\le\bar{\theta} \end{cases}.$$

3. *If* $\frac{q_H-p_H}{q_L-p_L}=1$, *all consumers apply to both types of service providers. For* $\theta \in \left[0,\bar{\theta}\right]$,

$$x_H(\theta)=x_L(\theta)=nf(\theta),$$
$$U(\theta)=(q_H-p_H)e^{-nF(\theta)}.$$

4. *If* $1<\frac{q_H-p_H}{q_L-p_L}<e^{\frac{n}{\gamma}}$, *consumers with type* $\theta \in [0,\theta_H]$ *apply to service providers of type H only, and consumers with* $\theta \in \left(\theta_H,\bar{\theta}\right]$ *apply to both types of service providers. For* $\theta \in \left[0,\bar{\theta}\right]$,

$$x_H(\theta)=\begin{cases} \frac{n}{\gamma}f(\theta), & 0\le\theta\le\theta_H \\ nf(\theta), & \theta_H<\theta\le\bar{\theta} \end{cases}, \quad x_L(\theta)=\begin{cases} 0, & 0\le\theta\le\theta_H \\ nf(\theta), & \theta_H<\theta\le\bar{\theta} \end{cases},$$
$$U(\theta)=\begin{cases} (q_H-p_H)e^{-\frac{n}{\gamma}F(\theta)}, & 0\le\theta\le\theta_H \\ (q_H-p_H)\left(\frac{q_H-p_H}{q_L-p_L}\right)^{-(1-\gamma)}e^{-nF(\theta)}, & \theta_H<\theta\le\bar{\theta} \end{cases}.$$

5. *If* $\frac{q_H-p_H}{q_L-p_L} \ge e^{\frac{n}{\gamma}}$, *all consumers apply to service providers of type H only. For* $\theta \in \left[0,\bar{\theta}\right]$,

$$x_H(\theta)=\frac{n}{\gamma}f(\theta), \quad x_L(\theta)=0,$$
$$U(\theta)=(q_H-p_H)e^{-\frac{n}{\gamma}F(\theta)}.$$

Figure 1 illustrates Theorem 1 intuitively. When q_H-p_H is significantly higher than q_L-p_L, all consumers apply to high-type service providers only, because the additional surplus from these providers more than compensates their lower acceptance rate. When $q_H - p_H$ is only slightly greater than $q_L - p_L$, only consumers with cost type θ below some threshold θ_H will apply to high-type service providers. Consumers with θ above the threshold θ_H are indifferent between the two types of service providers. When $q_H - p_H$ is equal to $q_L - p_L$, all consumers are indifferent between the two types of service providers, and apply to them with equal probability. Similarly, we can understand the cases when $q_H - p_H$ is smaller than $q_L - p_L$.

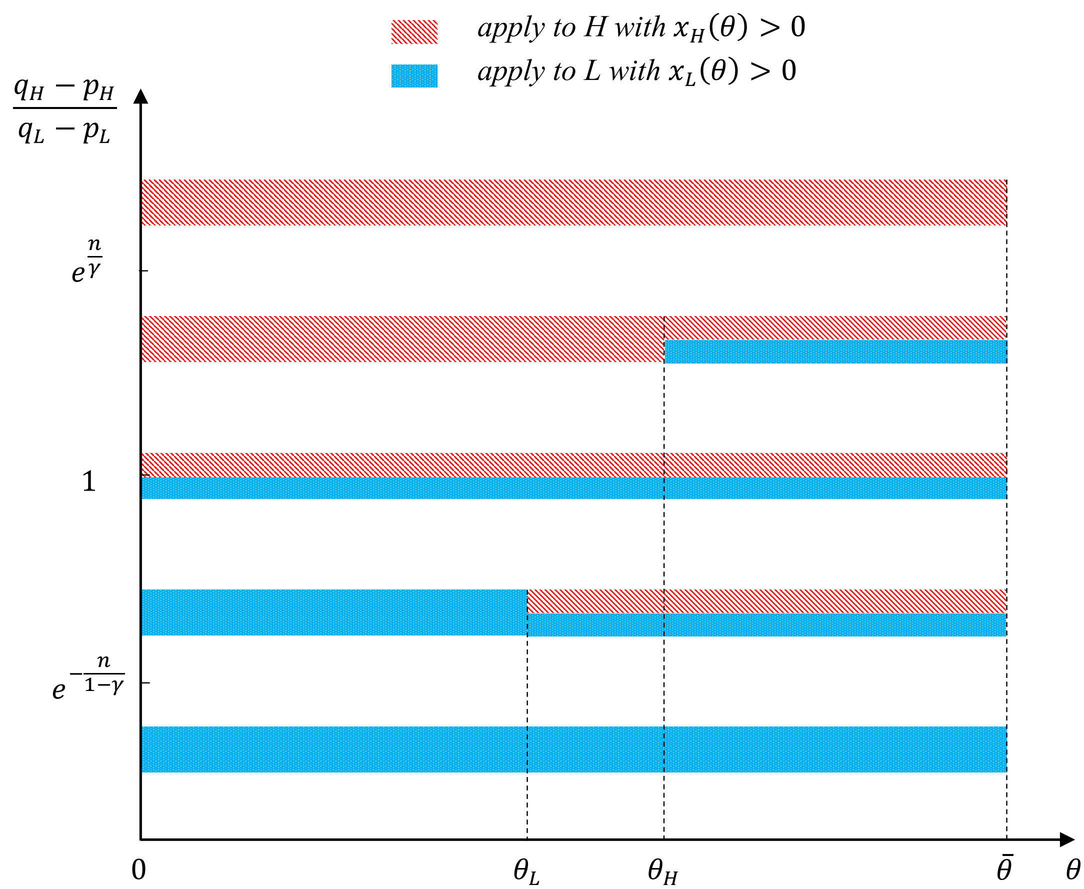

Figure 1: All potential market segmentations.

From a different perspective, Theorem 1 and Figure 1 also identify all possible market segmentations. One type of service providers could serve consumers in the entire cost spectrum, while the other type of providers serve a segment (subset) of consumers.

Lastly, it is worth noting that we have only considered consumers' application strategies when all service providers of the same type post the same price, i.e., symmetric pricing. When we consider an individual service provider's pricing decision as a next step, we have to consider his deviation of price with respect to other service providers in order to pin down the equilibrium. This leads to analysis of asymmetric prices. How can we utilize the consumers' application strategies derived only under symmetric pricing in Theorem 1 to inform an individual service provider's pricing decision? The key lies in the large market assumption that $M, N \to \infty$. Under this assumption, an individual service provider's deviation in price would not change individual consumers' maximum expected utility $U(\theta)$. We analyze service providers' pricing problem next.

2.2. Service Providers' Problem

Let us consider an individual service provider of type j, who posts price p_j^0 for $j \in \{H, L\}$. Given all other service providers' prices, p_H and p_L, $U(\theta)$ is completely determined by Theorem 1, which is taken as given by this individual service provider when choosing his price p_j^0. Now consider consumers' application strategy for this service provider. Given equation (3), we know that if $U(\theta) < q_j - p_j^0$, consumers of type θ apply to this service provider until $b_j^0(\theta)(q_j - p_j^0) = U(\theta)$, where $b_j^0(\theta)$ is this service provider's acceptance rate. On the other hand, if $U(\theta) \geq q_j - p_j^0$, consumers of type θ will not apply to this service provider with price p_j^0, because they have already got other more attractive service providers to apply to, who promise her a higher expected utility. From Theorem 1, we know that $U(\theta)$ is a strictly decreasing function. Therefore, we have,

$$b_j^0(\theta) = \frac{U(\theta)}{q_j - p_j^0} \text{ for } \theta \in [\theta^0, \bar{\theta}], \text{ where } \theta^0 = \begin{cases} 0, & q_j - p_j^0 \geq U(0), \\ U^{-1}(q_j - p_j^0), & \text{otherwise.} \end{cases} \tag{4}$$

Notice that by definition, $b_j^0(\theta)$ is the probability that the service provider has not received an application from consumers of types lower than θ. Correspondingly, $1 - b_j^0(\theta)$ is the probability that the service provider has received at least one application from consumers of types lower than θ. Therefore, $\mathrm{d}\left(1 - b_j^0(\theta)\right) = -\frac{\mathrm{d}b_j^0(\theta)}{\mathrm{d}\theta}\mathrm{d}\theta$ is the probability that the service provider has received at least one application from consumers of types in $[\theta, \theta + \mathrm{d}\theta]$. When a service provider receives no application, he ends up with no match and thus zero profit. When a service provider of type j receives and accepts an application from a consumer of type θ, he earns revenue p_j^0, pays the commission fee of δp_j^0 to the platform, and incurs serving cost of $\theta g(q_j)$. Therefore, we can write down this individual service provider's expected profit $\pi_j^0(p_j^0; p_H, p_L)$ as the following, given his price

p_j^0 as well as all other service providers' prices:

$$\begin{aligned}\pi_j^0(p_j^0; p_H, p_L) &= \int_{\theta^0}^{\bar{\theta}} \left[(1-\delta)p_j^0 \ - \theta g(q_j)\right] \cdot \left[-\frac{\mathrm{d}b_j^0(\theta)}{\mathrm{d}\theta}\right] \mathrm{d}\theta \\ &= \left[(1-\delta)p_j^0 - g(q_j)\theta^0\right] b_j^0(\theta^0) - \left[(1-\delta)p_j^0 - g(q_j)\bar{\theta}\right] b_j^0(\bar{\theta}) - g(q_j)\int_{\theta^0}^{\bar{\theta}} b_j^0(\theta)\mathrm{d}\theta \\ &= \begin{cases} -(1-\delta)\left[U(0) - U(\bar{\theta})\right] - \dfrac{\int_0^{\bar{\theta}} \left[(1-\delta)q_j - g(q_j)\theta\right] U'(\theta) d\theta}{q_j - p_j^0}, & p_j^0 \le q_j - U(0), \\ (1-\delta)U(\bar{\theta}) + (1-\delta)p_j^0 - g(q_j)U^{-1}(q_j - p_j^0) & \\ \quad - \dfrac{(1-\delta)U(\bar{\theta})q_j + g(q_j)\left[\int_{U^{-1}(q_j - p_j^0)}^{\bar{\theta}} U(\theta)\mathrm{d}\theta - \bar{\theta}U(\bar{\theta})\right]}{q_j - p_j^0}, & \text{otherwise.} \end{cases}\end{aligned} \tag{5}$$

The second equality above is due to integration by part and rearrangement of terms; to get the third equality, we have used $b_j^0(\theta)$ in equation (4). In writing down the profit function $\pi_j^0(p_j^0; p_H, p_L)$ above, we have assumed that the consumer market is fully covered: the service provider can make nonnegative profit even from the most costly consumer, i.e., $p_j^0 \ge \bar{\theta}g(q_j)/(1-\delta)$. We verify this assumption in equilibrium below and discuss alternative assumptions in Section 5.

The service provider's objective is to maximize the expected profit $\pi_j^0(p_j^0; p_H, p_L)$ by choosing the posted price p_j^0. In equilibrium, we must have $p_j^0 = p_j$. A pure strategy Nash equilibrium (p_H^*, p_L^*) is determined by

$$p_j^* = \arg\max_{p_j^0} \pi_j^0(p_j^0; p_H^*, p_L^*), \text{ for } j \in \{H, L\}. \tag{6}$$

As with most competitive search models in the literature (e.g., Rogerson et al. 2005), there is no closed-form expression for the equilibrium prices (p_H^*, p_L^*). We analyze the equilibrium as follows.

We first notice from equation (5) that $\pi_j^0(p_j^0; p_H, p_L)$ increases with p_j^0 when $p_j^0 \le q_j - U(0)$.[5] Therefore, we only need to consider the case that $p_j^0 \ge q_j - U(0)$. Correspondingly, $\pi_j^0(p_j^0; p_H, p_L)$ is given by the second case in equation (5). The optimal solution to the optimization problem in equation (6) must be either a corner solution with $p_j^0 = q_j - U(0)$ or an interior point with $p_j^0 > q_j - U(0)$.[6]

Meanwhile, from Theorem 1, we know that $U(0) = \max\{q_H - p_H, q_L - p_L\}$. Suppose in equilibrium, $q_H - p_H^* > q_L - p_L^*$. In this case, $U(0) = q_H - p_H^*$. Then the maximizing point of $\pi_H^0(p_H^0; p_H, p_L)$ will be the corner solution of $p_H^0 = q_H - U(0)$, which indeed is equal to p_H^*, and

[5]This is because $U'(\theta) < 0$ and $(1-\delta)q_j - g(q_j)\theta > (1-\delta)p_j^0 - g(q_j)\bar{\theta} \ge 0$.

[6]Notice that the corner solution of $p_j^0 = q_j - U(0)$ is possible, because $\pi_j^0(p_j^0; p_H, p_L)$ is continuous at $p_j^0 = q_j - U(0)$ and $\partial_{p_j^0}\pi_j^0(p_j^0; p_H, p_L)$ jumps by $-(1-\delta)U(0)p_j^0/(q_j - p_j^0)^2 < 0$ when p_j^0 increases from $(q_j - U(0))^-$ to $(q_j - U(0))^+$.

the maximizing point of $\pi_L^0(p_L^0; p_H, p_L)$ will be the interior solution of p_L^*, which indeed satisfies that $p_L^* > q_L - U(0) = q_L - (q_H - p_H^*)$. Similarly, we can analyze the other two cases with $q_H - p_H^* < q_L - p_L^*$ and $q_H - p_H^* = q_L - p_L^*$. The first-order necessary condition of optimality for the optimization problem in equation (6) can be summarized as the following,

$$\begin{cases} q_H - p_H^* < q_L - p_L^* \\ (1-\delta)(q_H - p_H^*)^2 - \left[(1-\delta)q_H - g(q_H)\bar{\theta}\right] U(\bar{\theta}) - g(q_H)\displaystyle\int_{\theta_L}^{\bar{\theta}} U(\theta)\mathrm{d}\theta = 0 \\ (1-\delta)(q_L - p_L^*)^2 - \left[(1-\delta)q_L - g(q_L)\bar{\theta}\right] U(\bar{\theta}) - g(q_L)\displaystyle\int_{0}^{\bar{\theta}} U(\theta)\mathrm{d}\theta \leq 0 \end{cases}, \text{ or} \tag{7}$$

$$\begin{cases} q_H - p_H^* = q_L - p_L^* \\ (1-\delta)(q_H - p_H^*)^2 - \left[(1-\delta)q_H - g(q_H)\bar{\theta}\right] U(\bar{\theta}) - g(q_H)\displaystyle\int_{0}^{\bar{\theta}} U(\theta)\mathrm{d}\theta \leq 0 \\ (1-\delta)(q_L - p_L^*)^2 - \left[(1-\delta)q_L - g(q_L)\bar{\theta}\right] U(\bar{\theta}) - g(q_L)\displaystyle\int_{0}^{\bar{\theta}} U(\theta)\mathrm{d}\theta \leq 0 \end{cases}, \text{ or} \tag{8}$$

$$\begin{cases} q_H - p_H^* > q_L - p_L^* \\ (1-\delta)(q_H - p_H^*)^2 - \left[(1-\delta)q_H - g(q_H)\bar{\theta}\right] U(\bar{\theta}) - g(q_H)\displaystyle\int_{0}^{\bar{\theta}} U(\theta)\mathrm{d}\theta \leq 0 \\ (1-\delta)(q_L - p_L^*)^2 - \left[(1-\delta)q_L - g(q_L)\bar{\theta}\right] U(\bar{\theta}) - g(q_L)\displaystyle\int_{\theta_H}^{\bar{\theta}} U(\theta)\mathrm{d}\theta = 0 \end{cases}. \tag{9}$$

$U(\theta)$ in equations (7), (8) and (9) is given in cases 2, 3, and 4 respectively in Theorem 1. Cases 1 and 5 in Theorem 1 never occur in equilibrium, because in these cases, all consumers apply only to one type of service providers. Consequently, the other type of service providers get zero profit. It is of each of these individual service providers' interest to decrease his price until either case 2 or 4 is satisfied. Again, in writing down equations (7)-(9), we have already utilized the expressions of $U(\theta)$ in Theorem 1. Particularly, in equation (7), $U^{-1}(q_L - p_L^*) = \theta_H$ according to case 2 in Theorem 1, and in equation (9), $U^{-1}(q_H - p_H^*) = \theta_L$ according to case 4 in Theorem 1.

We also require the profit function $\pi_j^0(p_j^0; p_H, p_L)$ in equation (5) to be concave in $p_j^0 \in [q_j - U(0), q_j]$ to ensure that the Nash equilibrium (p_H^*, p_L^*) exists. In the appendix, we prove that this is satisfied if and only if for $j \in \{H, L\}$,

$$
\begin{aligned}
nf(\theta) &\geq \left\{ \begin{array}{ll} \dfrac{(1-\gamma)U(\theta)}{2\left[\left(\frac{(1-\delta)q_j}{g(q_j)} - \bar{\theta}\right)U(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} U(t)\mathrm{d}t\right]}, & \text{for } \forall\theta \in [0, \theta_L] \\ \dfrac{U(\theta)}{2\left[\left(\frac{(1-\delta)q_j}{g(q_j)} - \bar{\theta}\right)U(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} U(t)\mathrm{d}t\right]}, & \text{for } \forall\theta \in [\theta_L, \bar{\theta}] \end{array} \right\} \text{if } q_H - p_H \leq q_L - p_L, \\
nf(\theta) &\geq \left\{ \begin{array}{ll} \dfrac{\gamma U(\theta)}{2\left[\left(\frac{(1-\delta)q_j}{g(q_j)} - \bar{\theta}\right)U(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} U(t)\mathrm{d}t\right]}, & \text{for } \forall\theta \in [0, \theta_H] \\ \dfrac{U(\theta)}{2\left[\left(\frac{(1-\delta)q_j}{g(q_j)} - \bar{\theta}\right)U(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} U(t)\mathrm{d}t\right]}, & \text{for } \forall\theta \in [\theta_H, \bar{\theta}] \end{array} \right\} \text{if } q_H - p_H > q_L - p_L.
\end{aligned} \tag{10}
$$

This implies that we must impose a lower bound on $nf(\cdot)$ to ensure concavity of $\pi_j^0(p_j^0; p_H, p_L)$. The intuition is the following. For $\pi_j^0(p_j^0; p_H, p_L)$ to be concave, we require $\pi_j^0(p_j^0; p_H, p_L)$ to first increase and then decrease to zero as p_j^0 increases from 0 to q_j. Let us consider the interval of p_j^0 where $\pi_j^0(p_j^0; p_H, p_L)$ is supposed to decrease with p_j^0. Given a marginal increase in price p_j^0, the marginal number of consumers who switch from the deviating service provider to others is proportional to $nf(\theta^0)$, which cannot be too small. Otherwise, the service provider may be willing to give up these consumers. In other words, $\pi_j^0(p_j^0; p_H, p_L)$ may increase with p_j^0 again.

Equation (10) implies that a general distribution of consumers' cost types may lead to non-concave profit functions, which may in turn result in non-existence of a pure strategy Nash equilibrium. Therefore, to facilitate equilibrium analysis below, we restrict our attention to the uniform distribution only, where $F(\theta) = \theta/\bar{\theta}$. With uniform distribution, the concavity condition in equation (10) essentially imposes a lower bound on n.

3. The Equilibrium

In this section, we analyze the equilibrium based on the optimality conditions (7)-(9), and the concavity condition (10). We characterize the equilibrium by the following theorem.

Theorem 2: *Assume uniform distribution of $F(\cdot)$ and concavity condition (10).*

- *There always exist an infinite number of solutions to the problem in equation (6), which satisfy $q_H - p_H^* = q_L - p_L^* = \varepsilon$ for $\forall \varepsilon \in (0, \bar{\varepsilon}]$, where,*

$$\bar{\varepsilon} = \min_{j \in \{H,L\}} e^{-n} q_j + \frac{1 - (n+1)e^{-n}}{(1-\delta)n} \bar{\theta} g(q_j) \geq \frac{\bar{\theta} g(g_L)}{2(1-\delta)n} > 0.$$

- *In the case that $g(q)=1$ or $g(q)=q$, all equilibria satisfy that $q_H - p_H^* \geq q_L - p_L^*$.*

Theorem 2 shows that $(p_H^*, p_L^*) = (q_H - \varepsilon, q_L - \varepsilon)$ is always an equilibrium for ε positive but sufficiently small.[7] This parallels with the well-known Diamond Paradox (Diamond 1971), where competitive sellers can gain monopolistic power in a homogeneous product market with consumer search costs. To reiterate, in our model, market friction is not modeled by consumer search costs, but instead, by coordination frictions.

Theorem 2 also shows that with $g(q) = 1$ or $g(q) = q$, it never occurs in equilibrium that $q_H - p_H^* < q_L - p_L^*$. That is to say, in equilibrium, whenever a consumer is accepted, she expects the same or higher utility from service providers with higher quality. This implies that there are only two possible market segmentations in equilibrium, as shown by Figure 2: either all consumers apply to both types of service providers; or low-cost consumers apply solely to high-quality service providers and high-cost consumers apply to both types of service providers.

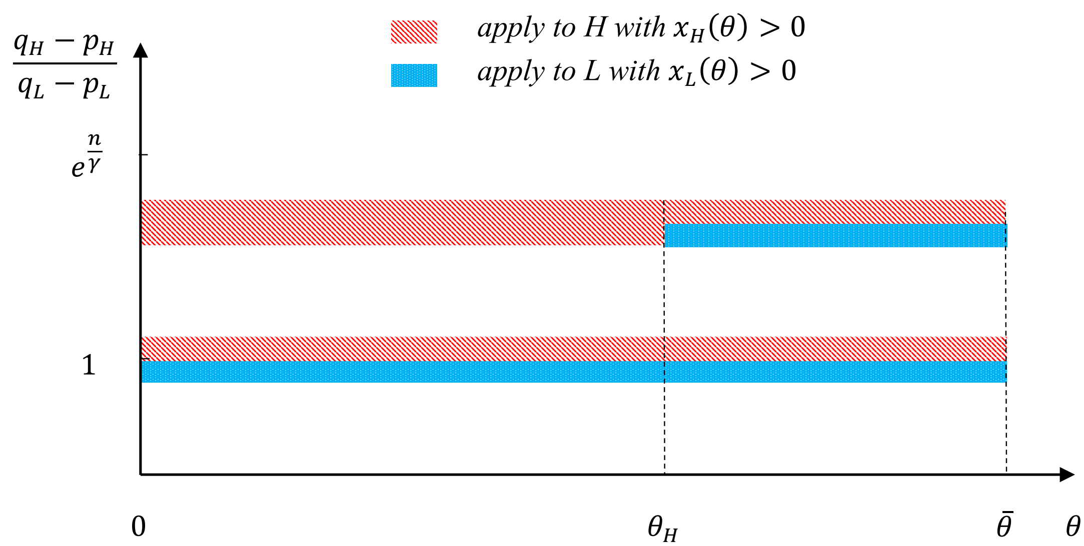

Figure 2: Equilibrium market segmentations.

Figure 3 shows all price equilibria under one parameter setting, with $n = 1$, $\gamma = 0.5$, $q_H = 2q_L$, $\delta = 0.1$, $\bar{\theta} = 0.2$, and $g(q) = 1$. We use the same parameter setting for the following analysis, and refer to it as “the parameter setting" for simplicity. We can see that under this parameter setting, both types of price equilibria are possible: there are infinite equilibria with $q_H - p_H^* = q_L - p_L^*$, and

[7] We do not consider the case with $\varepsilon = 0$, because in this case, consumers expect zero utility from all service providers. Consequently, each consumer is indifferent between the two types of service providers, and there is arbitrariness in consumers' application strategy.

there are also infinite equilibria with $q_H - p_H^* > q_L - p_L^*$. Moreover, compared with equilibria with $q_H - p_H^* = q_L - p_L^*$, equilibria with $q_H - p_H^* > q_L - p_L^*$ are associated with lower p_H^* and p_L^*.

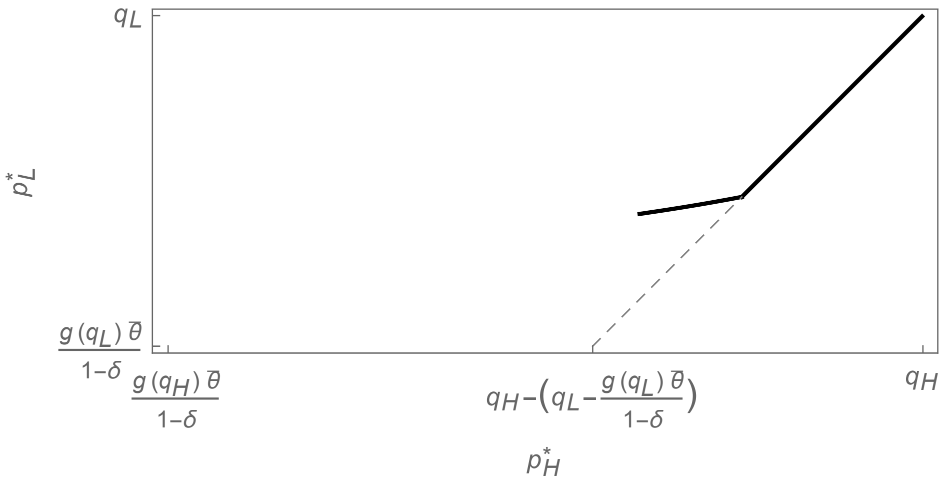

Figure 3: Equilibrium prices under the parameter setting that $n = 1$, $\gamma = 0.5$, $q_H = 2q_L$, $\delta = 0.1$, $\bar{\theta} = 0.2$, and $g(q) = 1$.

By combining Theorem 1 and Theorem 2, we characterize consumers' application strategies, acceptance rates, and expected utilities in the following corollary. Notice that we can unify the two types of price equilibria by noting that when $q_H - p_H = q_L - p_L$, $\theta_H = 0$ and $[0, \theta_H) = \emptyset$.

COROLLARY 1: *Assume the uniform distribution of $F(\cdot)$, concavity condition (10), and $g(q) = 1$ or $g(q) = q$. In equilibrium, consumers with type $\theta \in [0, \theta_H)$ apply to service providers of type H only, and consumers with $\theta \in [\theta_H, \bar{\theta}]$ apply to both types of service providers. The equilibrium prices satisfy that $q_H - p_H^* \geq q_L - p_L^*$. The equilibrium queue lengths, acceptance rates, and consumers' expected utilities, are respectively,*

$$x_H(\theta) = \begin{cases} \frac{n}{\gamma\bar{\theta}}, & 0 \leq \theta \leq \theta_H \\ \frac{n}{\bar{\theta}}, & \theta_H < \theta \leq \bar{\theta} \end{cases}, \quad x_L(\theta) = \begin{cases} 0, & 0 \leq \theta \leq \theta_H \\ \frac{n}{\bar{\theta}}, & \theta_H < \theta \leq \bar{\theta} \end{cases};$$

$$b_H(\theta) = \begin{cases} e^{-\frac{n\theta}{\gamma\bar{\theta}}}, & 0 \leq \theta < \theta_H \\ \left(\frac{q_H - p_H}{q_L - p_L}\right)^{-(1-\gamma)} e^{-\frac{n\theta}{\bar{\theta}}}, & \theta_H \leq \theta \leq \bar{\theta} \end{cases}, \quad b_L(\theta) = \begin{cases} 1, & 0 \leq \theta < \theta_H \\ \left(\frac{q_H - p_H}{q_L - p_L}\right)^{\gamma} e^{-\frac{n\theta}{\bar{\theta}}}, & \theta_H \leq \theta \leq \bar{\theta} \end{cases};$$

$$U(\theta) = \begin{cases} (q_H - p_H)e^{-\frac{n\theta}{\gamma\bar{\theta}}}, & 0 \leq \theta \leq \theta_H \\ (q_H - p_H)\left(\frac{q_H - p_H}{q_L - p_L}\right)^{-(1-\gamma)} e^{-\frac{n\theta}{\bar{\theta}}}, & \theta_H < \theta \leq \bar{\theta} \end{cases}.$$

Figure 4 plots the consumers' application strategy (queue lengths), acceptance rates, as well as their expected utility in both types of equilibria. In the equilibrium with $q_H - p_H^* = q_L - p_L^*$, two types of service providers are of the same attractiveness to consumers, so consumers' equilibrium queue lengths and acceptance rates are exactly the same for two types of service providers. In contrast, in the equilibrium with $q_H - p_H^* > q_L - p_L^*$, high-quality service providers are more attractive and thus are more competitive among consumers. Consequently, the queue length for high-quality service providers is longer than that for low-quality providers for low-cost consumers and the acceptance rate for high-quality service providers is lower for all types of consumers.

Finally, we notice that for both types of equilibria, consumers with higher costs expect lower utility. In general, it is straightforward to prove that $U(\theta)$ decreases with θ. More interestingly, if we compare the expected utility of a consumer of cost type θ, $U(\theta)$ with that of a consumer of cost type 0, $U(0)$. We find that $U(\theta) < U(0) - \theta$, i.e., given all other agents' strategy in equilibrium, a consumer is willing to reimburse service providers of her serving cost so as to let herself be treated as if her serving cost were zero. In other words, a consumer with higher serving cost is subject to disproportionately lower acceptance rate in equilibrium.

3.1. Comparative Statics

Given the existence of multiple equilibria, it is difficult if not infeasible to analytically compute comparative statics; instead, we plot the comparative statics numerically in Figure 5 below.

Let us understand the intuition behind the six plots in Figure 5. As the ratio of consumers to service providers, n, gets larger, the service providers' market becomes less competitive, so the equilibrium prices can get higher. As the fraction of high-quality service providers, γ, gets larger, the high-quality service providers' market becomes more competitive, and their price p_H^* can get lower. As a result, the low-quality service providers' price also gets lower. This happens despite that low-quality service providers' own market gets less competitive. In essence, the increase in competition from high-quality service providers dominates the reduction in competition among low-quality providers themselves, so low-quality providers' prices will also decrease.

As consumers' costs to serve, $\bar{\theta}$ increase, the equilibrium prices can surprisingly get lower. This is in stark contrast to production costs, which usually have a positive pass-through to prices. The intuition is as follows. As an individual service provider raises his price, he loses applications from consumers with low serving costs, but gains applications from those with high serving costs. Particularly, from equation (4) and (5), we find that the demand from consumer θ, $-\frac{\mathrm{d}b_j^0(\theta)}{\mathrm{d}\theta}\mathrm{d}\theta$ increases with the type-j service provider's price p_j^0 for relatively small θ and increases with p_j^0 for relatively large θ. When $\bar{\theta}$ gets higher, a service provider decreases his price to reduce demand from

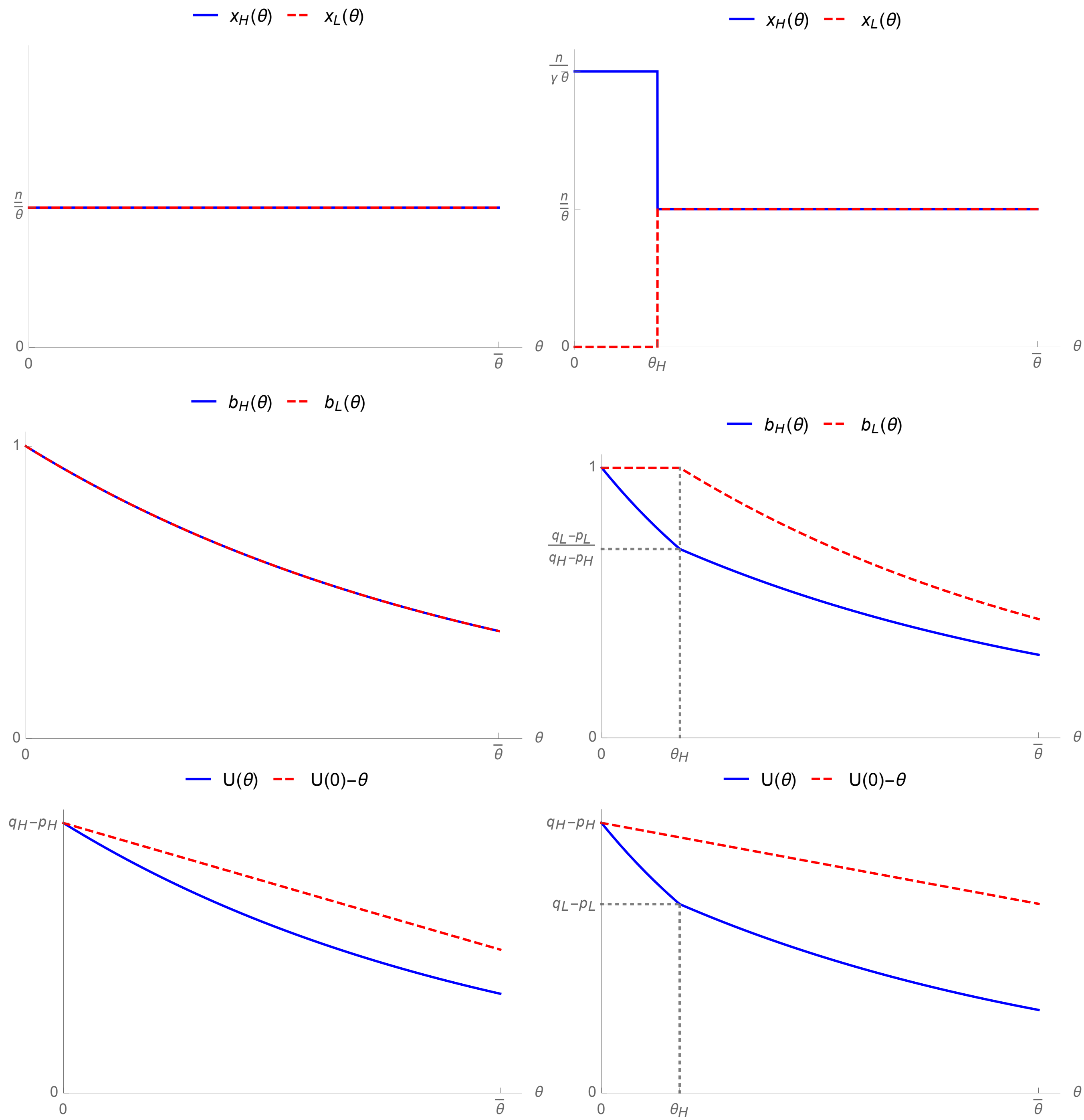

Figure 4: Equilibrium queue lengths, acceptance rates, and consumers' expected utilities, under the same parameter setting as with Figure 3. In the left panels, $(p_H^*, p_L^*) = (0.787q_H, 0.573q_L)$ with $q_H - p_H^* = q_L - p_L^*$; in the right panels, $(p_H^*, p_L^*) = (0.666q_H, 0.533q_L)$ with $q_H - p_H^* > q_L - p_L^*$.

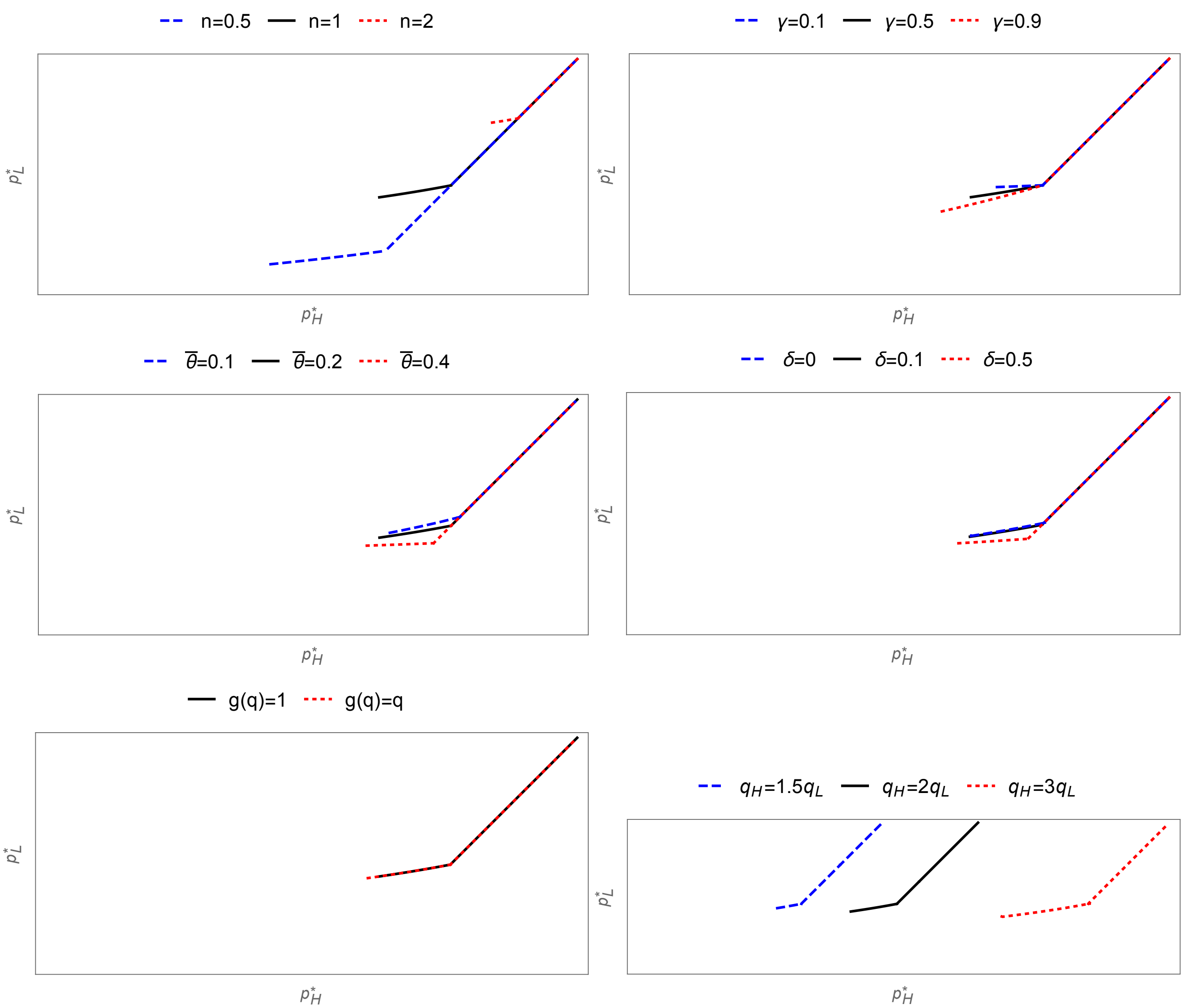

Figure 5: Equilibrium prices under the same parameter setting as with Figure 3 except for the parameter of interest.

high-cost consumers and increase demand from low-cost consumers. As a result, the equilibrium price can get lower due to providers' enhanced incentive to get better consumers. To put in another word, in our model, the composition of consumers faced by each service provider is endogenous—it depends on the service provider's price and others' prices. By setting a lower price, a service provider can get more consumer applications, which enables him to cherrypick a low-cost consumer. This incentive to lower price becomes stronger, when consumers' serving costs get higher. With similar intuition, one can see that as the matching platform's commission fee (δ) increases, the equilibrium prices can decrease.

Compared with the case $g(q) = 1$, we find that the equilibrium prices with $g(q) = q$ are lower. This is because when $g(q) = q$, high-quality service providers find it more costly to serve high-cost consumers, which leads them to lower their prices to induce applications from the lowest-cost customers. Consequently, low-quality service providers also have to lower their prices given the increased competition from high-quality providers. Lastly, as service quality q_H gets higher, the equilibrium price p_H^* gets higher, which is intuitive.

4. Comparison with Unilateral Ratings

In this section, we consider the unilateral rating system, where only service providers' quality is publicly observable and consumers' cost information is not. With unilateral ratings, service providers cannot discern consumers with low costs from those with high costs, so they will randomly select a consumer when receiving multiple applications. This is obviously the extreme case, which does not correspond to most real-world applications. Nevertheless, our objective is to understand how information disclosure changes the market structure, and unilateral rating is the natural setting to consider. As before, we consider the case of a large market with $N, M \to \infty$, and focus on symmetric equilibria.

Denote the probability that a consumer submits an application to a service provider of type j by A_j, for $j \in \{H, L\}$ and the queue length at a service provider of type j by $X_j = NA_j$. The normalization condition $\gamma MA_H + (1 - \gamma)MN_L = 1$ can be rewritten in terms of X_H and X_L as

$$\gamma X_H + (1 - \gamma)X_L = n. \tag{11}$$

Given the service providers' prices, p_H and p_L, a consumer's acceptance rate by a service provider of type j conditioning on that the consumer has submitted her application to the service provider, is,

$$B_j = \lim_{N,M\to\infty} \sum_{i=0}^{N-1} \binom{N-1}{i} A_j^i (1-A_j)^{N-1-i} \frac{1}{i+1} = \lim_{N_j,M\to\infty} \frac{1-(1-A_j)^N}{NA_j} = \frac{1-e^{-X_j}}{X_j}. \quad (12)$$

In equilibrium, a consumer's expected utility from the two types of service providers should be the same, denoted as U. Otherwise, consumers would deviate to applying to service providers with higher expected utility, which lowers their acceptance rate until the expected utilities of the two types are equal. Therefore, we have $U = B_H(q_H - p_H) = B_L(q_L - p_L)$, i.e.,

$$U = (q_H - p_H)\frac{1-e^{-X_H}}{X_H} = (q_L - p_L)\frac{1-e^{-X_L}}{X_L}. \quad (13)$$

By combining equations (11) and (13), we can in theory solve X_H and X_L and thus determine U. Consider now an individual service provider j's profit maximization problem. Given his posted price p_j^0, his acceptance rate is

$$B_j^0 = \min\left\{\frac{U}{q_j - p_j^0}, 1\right\}. \quad (14)$$

Define function $\phi(x) \equiv [1-e^{-x}]/x$ for $x \in (0,\infty)$ and $\phi(0) \equiv \lim_{x\to 0}\phi(x) = 1$. We know that $\phi(x)$ is a continuous and strictly decreasing function on $[0,\infty)$. Given the service provider's acceptance rate B_j^0, his queue length is $X_j^0 = \phi^{-1}(B_j^0)$. The service provider's expected profit, given his posted price p_j^0 and the market prices p_H and p_L, will be

$$\begin{aligned}\Pi_j^0(p_j^0; p_H, p_L) &= \left[(1-\delta)p_j^0 - \frac{\bar{\theta}}{2}g(q_j)\right] X_j^0 B_j^0 \\ &= \begin{cases} \left[(1-\delta)p_j^0 - \frac{\bar{\theta}}{2}g(q_j)\right]\left(\frac{U}{q_j - p_j^0}\right)\phi^{-1}\left(\frac{U}{q_j - p_j^0}\right), & \text{if } p_j^0 \le q_j - U, \\ 0, & \text{otherwise.}\end{cases}\end{aligned} \quad (15)$$

Based on equation (15), to ensure that the service provider has a nonnegative profit margin, we must have $p_j^0 \ge \bar{\theta}g(q_j)/[2(1-\delta)]$; to ensure that the service provider has positive demand, we must have $p_j^0 \le q_j - U$. Therefore, we must have $\bar{\theta}g(q_j)/[2(1-\delta)] \le q_j - U$ to ensure that the service provider is willing to participate in the market. We verify this condition in equilibrium, which is similar to the full market coverage condition in the case of bilateral ratings. Given the participation constraint, the service provider only considers $p_j^0 \in \left[\bar{\theta}g(q_j)/[2(1-\delta)], q_j - U\right]$, because $\Pi_j^0\left(\bar{\theta}g(q_j)/[2(1-\delta)]; p_H, p_L\right) = \Pi_j^0(q_j - U; p_H, p_L) = 0$. In the appendix, we show that $\Pi_j^0(p_j^0; p_H, p_L)$ is concave in p_j^0, so there exists a unique solution to the first-order optimality condition: $\partial\Pi_j^0(p_j^0; p_H, p_L)/\partial p_j^0|_{p_j^0=p_j} = 0$, which can be written as

$$\frac{q_j - p_j}{U} - \frac{(1-\delta)(q_j - p_j)}{(1-\delta)q_j - \frac{\bar{\theta}}{2}g(q_j)} = \ln\left(\frac{q_j - p_j}{U}\right) - \ln\left[\frac{(1-\delta)(q_j - p_j)}{(1-\delta)q_j - \frac{\bar{\theta}}{2}g(q_j)}\right] \geq 0,\ j \in \{H, L\}. \quad (16)$$

The equilibrium is the set of (p_H, p_L, X_H, X_L, U) that satisfies equations (11), (13) and (16). There are five equations in total to determine five unknown variables. In general, when the five equations are non-degenerate, we have a unique equilibrium solution of (p_H, p_L, X_H, X_L, U). The following theorem characterizes the existence and some properties of the equilibrium.

THEOREM 3: *With unilateral ratings, when equations (11), (13) and (16) are not degenerate, there exists a unique pure strategy Nash equilibrium* (p_H^{**}, p_L^{**}) *to the service providers' pricing game. In the case that* $q_H > q_L$*, and* $g(q) = 1$ *or* $g(q) = q$*, we have* $q_H - p_H^{**} > q_L - p_L^{**}$*.*

It is difficult to compare the equilibria of unilateral and bilateral ratings analytically, because we do not have closed-form expressions for equilibrium prices for both cases. We resort to numeric examples instead. Figure 6 plots the price equilibrium with both bilateral and unilateral ratings. With unilateral ratings, there is indeed a unique equilibrium. What is more interesting is that compared with unilateral ratings, bilateral ratings *may* lead to higher equilibrium prices. This is because bilateral ratings facilitate market segmentation and can thus soften the competition among service providers. We solve the equilibrium for all parameter settings in the comparative statics in Figure 5 and find that equilibrium prices with bilateral ratings are always higher than those with unilateral ratings.

Although equilibrium prices under bilateral ratings are higher, one may suspect that consumer surplus under bilateral ratings can still be higher because consumers get sorted according to their costs and low-cost consumers can benefit from a high acceptance rate. In Figure 7, however, we show that bilateral ratings can lower total consumer surplus. Consumer surplus is represented by the area below the curve $U(\theta)$ in the case of bilateral ratings and U in the case of unilateral ratings. We find that for consumers with low serving costs, indeed, bilateral ratings can increase their surplus because of the higher acceptance rates; for consumers with high serving costs, however, bilateral ratings decrease their surplus because of higher prices. Overall, in this example, bilateral ratings decreases the total consumer surplus. We find that in other parameter settings, it is possible that the consumer surplus is higher under bilateral ratings, when the low-cost consumers' welfare gains dominate the high-cost consumers' welfare loss. In the peer-to-peer market, we should also be cautious about the definition of consumer surplus, because service providers can also be individual consumers. In this

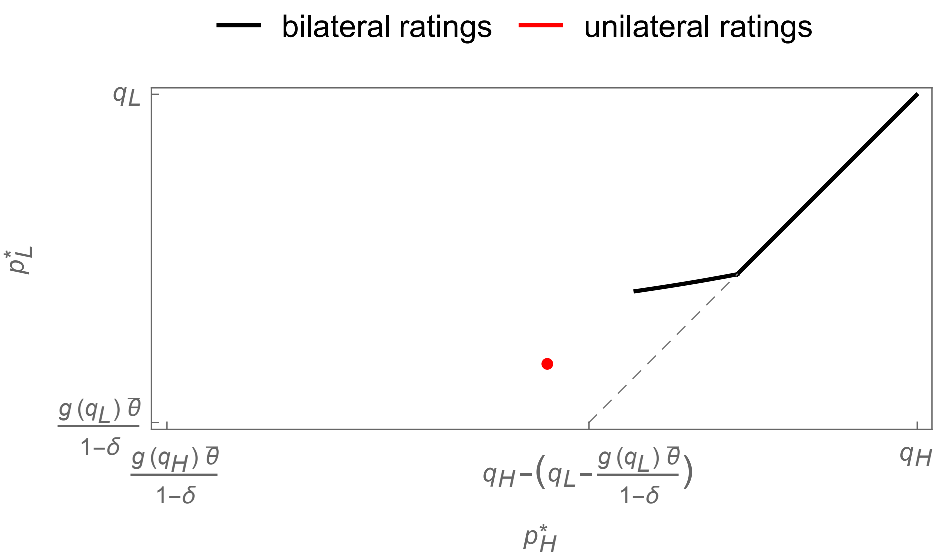

Figure 6: Equilibrium prices with bilateral and unilateral ratings, under the same parameter setting of Figure 3. The red dot is the equilibrium under unilateral ratings, and the black line is the equilibria under bilateral ratings.

paper, when calculating consumer surplus, we only count the surplus of "consumers" not including service providers' surplus.

In Figure 8, we present the full decomposition of social welfare under one parameter setting. We find that, bilateral ratings can increase both high- and low-quality service providers' profits, as well as the platform's profit, at the cost of consumer surplus. The total social welfare is slightly higher in the case of bilateral ratings in this example. In the case with $g(q) = 1$, the total surplus entirely depends on the number of matches. We find that in other parameter settings, it is possible that the total surplus is lower under bilateral ratings.

To summarize the findings in this section, we have compared the market structures between bilateral and unilateral ratings. A numerically robust finding is that bilateral ratings lead to higher prices than unilateral ratings, because bilateral ratings facilitate market segmentation and thus soften the price competition. The welfare implications are not a clear-cut—bilateral ratings could lead to either higher or lower total surplus as well as consumer surplus.

5. Incomplete Market Coverage

In this section, we consider the case of incomplete market coverage: Some consumers can be so costly that it is not profitable for some service providers to serve them. Particularly, we consider

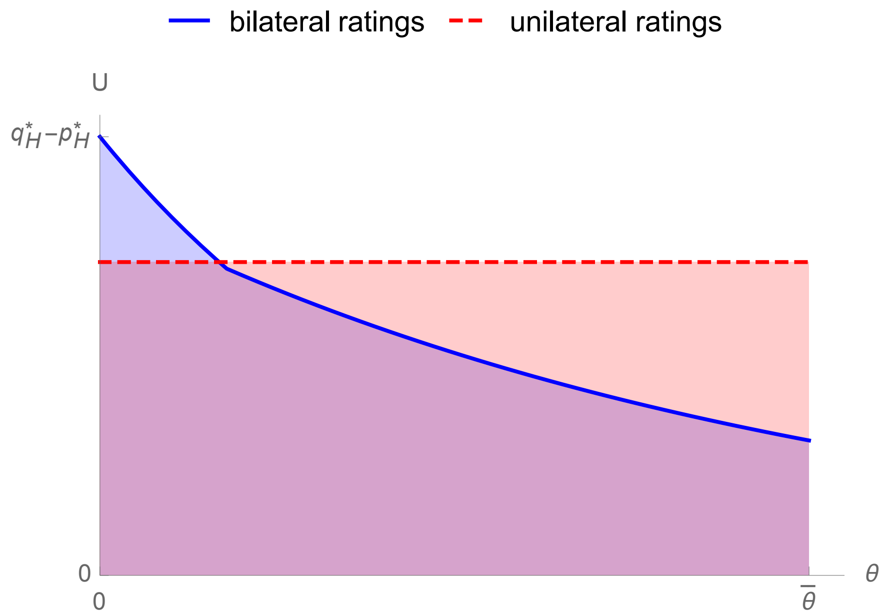

Figure 7: Comparison of consumer surplus under the same parameter setting of Figure 3.

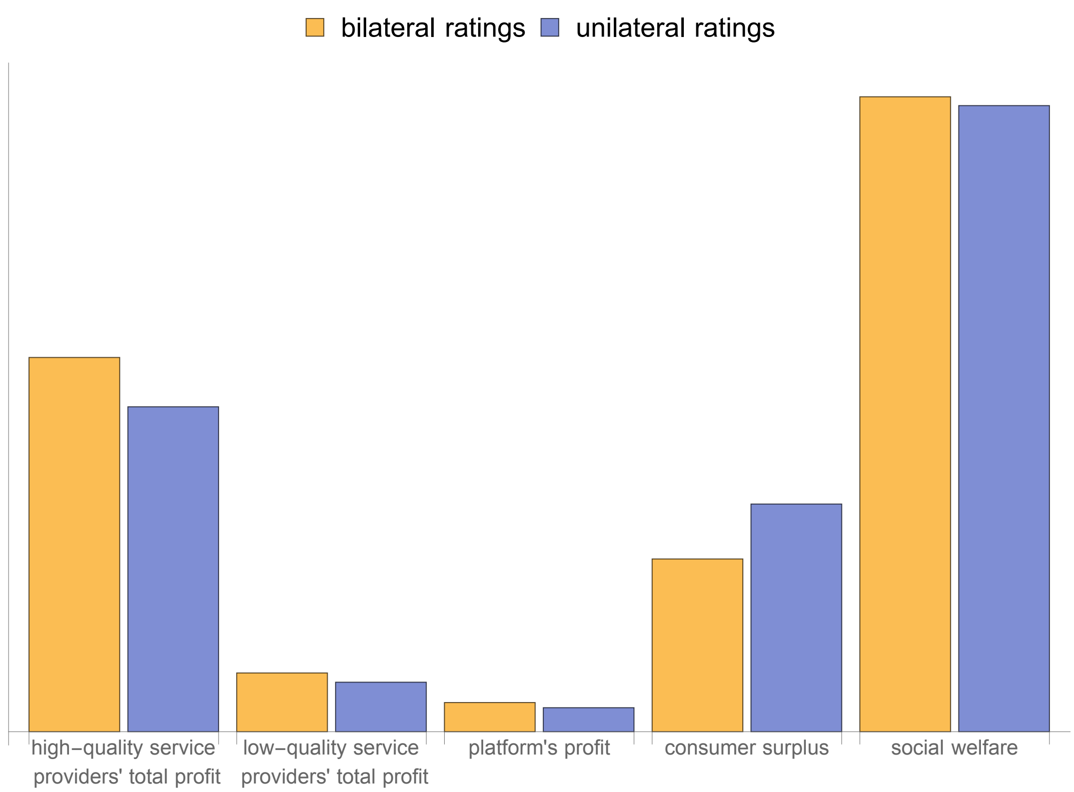

Figure 8: Comparison of welfare breakdowns, under the same parameter setting of Figure 3.

the most interesting case that the market coverage is complete for low-quality service providers but incomplete for high-quality service providers, i.e.,

$$p_H < \bar{\theta} g(q_H)/(1-\delta) \text{ and } p_L \geq \bar{\theta} g(q_L)/(1-\delta). \tag{17}$$

This setting corresponds to some real-world circumstances we observe in peer-to-peer markets, such as Airbnb. Owners of high-end apartments may lose money by hosting some nasty guests, while owners of inexpensive apartments may have little to lose.

We consider both cases $g(q) = 1$ and $g(q) = q$ below. In the case of $g(q) = 1$, condition (17) implies that $p_H < \bar{\theta}/(1-\delta) \leq p_L$, which implies that $q_H - p_H > q_L - p_L$. In the case of $g(q) = q$, condition (17) implies that $0 \leq q_L - p_L \leq q_L \left[1 - \bar{\theta}/(1-\delta)\right]$, which implies that $\bar{\theta} + \delta \leq 1$. Under this condition, we have that $q_H - p_H > q_H \left[1 - \bar{\theta}/(1-\delta)\right] \geq q_L \left[1 - \bar{\theta}/(1-\delta)\right] \geq q_L - p_L$. Therefore, the incomplete market coverage condition in equation (17) implies that $q_H - p_H > q_L - p_L$ for both cases that $g(q) = q$ and $g(q) = 1$.

Similar to Theorem 1, we characterize consumers' application strategies given service providers' posted prices p_H and p_L in the following theorem. The proof is similar and thus omitted.

THEOREM 4: *Under the incomplete market condition (17) and $\bar{\theta} + \delta \leq 1$, we have $q_H - p_H > q_L - p_L$.*

1. *If $\frac{q_H - p_H}{q_L - p_L} < e^{\frac{n}{\gamma} F\left(\frac{(1-\delta)p_H}{g(q_H)}\right)}$, consumers with type $\theta \in [0, \theta_H]$ apply to service providers of type H only, consumers with $\theta \in (\theta_H, (1-\delta)p_H/g(q_H)]$ apply to both types of service providers, and consumers with $\theta \in \left(1-\delta)p_H/g(q_H), \bar{\theta}\right]$ apply to service providers of type L only. For $\theta \in \left[0, \bar{\theta}\right]$,*

$$x_H(\theta) = \begin{cases} \frac{n}{\gamma} f(\theta), & 0 \leq \theta \leq \theta_H \\ n f(\theta), & \theta_H < \theta \leq \frac{(1-\delta)p_H}{g(q_H)} \\ 0, & \frac{(1-\delta)p_H}{g(q_H)} < \theta \leq \bar{\theta} \end{cases}, \quad x_L(\theta) = \begin{cases} 0, & 0 \leq \theta \leq \theta_H \\ n f(\theta), & \theta_H < \theta \leq \frac{(1-\delta)p_H}{g(q_H)} \\ \frac{n}{1-\gamma} f(\theta), & \frac{(1-\delta)p_H}{g(q_H)} < \theta \leq \bar{\theta} \end{cases},$$

$$U(\theta) = \begin{cases} (q_H - p_H) e^{-\frac{n}{\gamma} F(\theta)}, & 0 \leq \theta \leq \theta_H \\ (q_H - p_H) \left(\frac{q_H - p_H}{q_L - p_L}\right)^{-(1-\gamma)} e^{-nF(\theta)}, & \theta_H < \theta \leq \frac{(1-\delta)p_H}{g(q_H)} \\ (q_H - p_H) \left(\frac{q_H - p_H}{q_L - p_L}\right)^{-(1-\gamma)} e^{\frac{\gamma}{1-\gamma} nF\left(\frac{(1-\delta)p_H}{g(q_H)}\right)} e^{-\frac{1}{1-\gamma} nF(\theta)}, & \frac{(1-\delta)p_H}{g(q_H)} < \theta \leq \bar{\theta} \end{cases}.$$

2. *Otherwise, if $\frac{q_H - p_H}{q_L - p_L} \geq e^{\frac{n}{\gamma} F\left(\frac{(1-\delta)p_H}{g(q_H)}\right)}$, consumers with type $\theta \in [0, (1-\delta)p_H/g(q_H)]$ apply to service providers of type H only, and consumers with $\theta \in \left(1-\delta)p_H/g(q_H), \bar{\theta}\right]$ apply to service*

providers of type L only. For $\theta \in [0, \bar{\theta}]$,

$$x_H(\theta) = \begin{cases} \frac{n}{\gamma} f(\theta), & 0 \le \theta \le \frac{(1-\delta)p_H}{g(q_H)} \\ 0, & \frac{(1-\delta)p_H}{g(q_H)} < \theta \le \bar{\theta} \end{cases}, \quad x_L(\theta) = \begin{cases} 0, & 0 \le \theta \le \frac{(1-\delta)p_H}{g(q_H)} \\ \frac{n}{1-\gamma} f(\theta), & \frac{(1-\delta)p_H}{g(q_H)} < \theta \le \bar{\theta} \end{cases},$$

$$U(\theta) = \begin{cases} (q_H - p_H) e^{-\frac{n}{\gamma} F(\theta)}, & 0 \le \theta \le \frac{(1-\delta)p_H}{g(q_H)} \\ (q_L - p_L) e^{\frac{1}{1-\gamma} n F\left(\frac{(1-\delta)p_H}{g(q_H)}\right)} e^{-\frac{1}{1-\gamma} n F(\theta)}, & \frac{(1-\delta)p_H}{g(q_H)} < \theta \le \bar{\theta} \end{cases}.$$

The big difference, when compared with the main model of complete market coverage, is that consumers with cost type $\theta \in \left((1-\delta)p_H/g(q_H), \bar{\theta}\right]$ may prefer to apply to high-quality service providers. However, high-quality service providers would refuse to serve them, because the marginal benefit does not offset the serving cost. These consumers therefore have no other choice but to apply to low-quality service providers. As a result, in the second case of Theorem 4 when $\frac{q_H - p_H}{q_L - p_L} \ge e^{\frac{n}{\gamma} F\left(\frac{(1-\delta)p_H}{g(q_H)}\right)}$, $U(\theta)$ is not continuous—it jumps downward at $\theta = (1-\delta)p_H/g(q_H)$. Figure 9 characterizes the resulting market segmentations. We find that under incomplete market coverage, market segmentation could become even more complicated, with low-cost consumers applying to high-quality service providers only, high-cost consumers applying to low-quality service providers only, and medium-cost consumers applying to both.

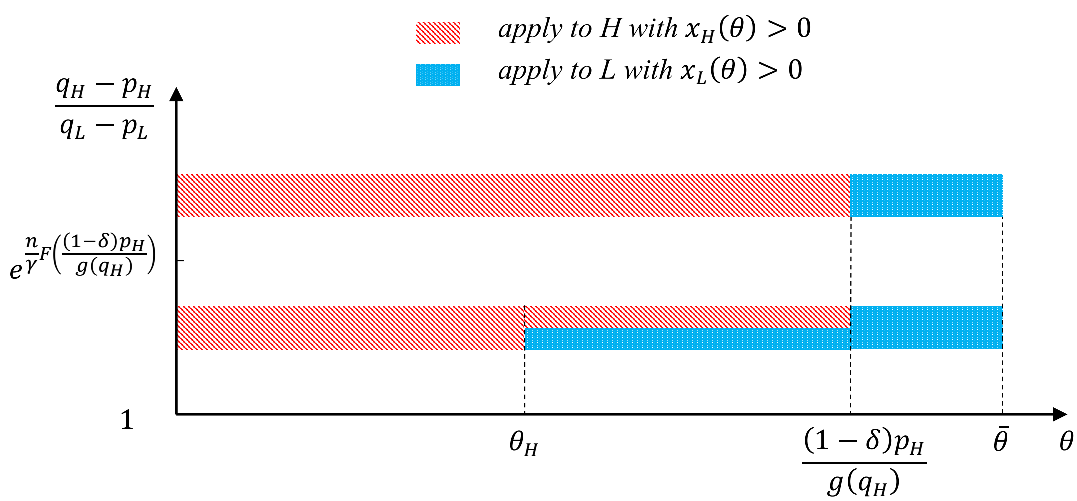

Figure 9: All potential market segmentations under incomplete market coverage.

Given consumers' application strategies and expected utilities in Theorem 4, we can consider an individual service provider's pricing decision along similarly as in our main model. We relegate the details in Appendix.

Instead of fully characterizing the equilibrium, which is quite complicated, we are interested in one particular property of the equilibrium. Figure 10 demonstrates a specific parameter setting to illustrate our idea. This is the setting where the range of consumers' serving costs is large with $\bar{\theta} = 0.8$, and the fraction of high-quality service providers is large with $\gamma = 0.9$. In this case, competition among high-quality service providers drives down their equilibrium price, to the degree that *the equilibrium price of high-quality service providers can be lower than that of low-quality service providers*, as shown by the solid line in Figure 10. This occurs because consumers with $\theta \in ((1-\delta)p_H/g(q_H), \bar{\theta}]$ are too costly for high-quality service providers to serve. Low-quality service providers face relatively less competition in serving these high-cost consumers, and charge a high price to cover the high serving costs. On the other hand, high-quality service providers charge a low price to cherrypick the low-cost consumers to serve. This is in stark contrast with the price competition outcomes in traditional vertically differentiated markets, where high-quality sellers always charge a higher price.

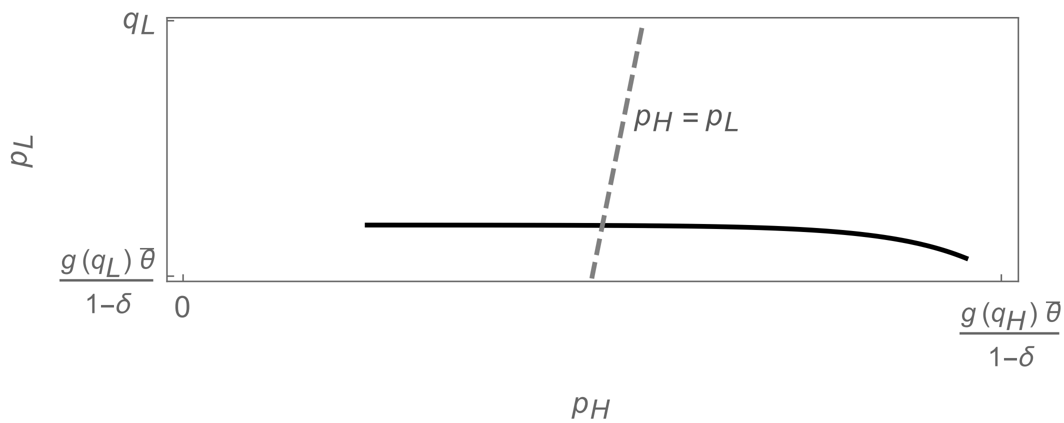

Figure 10: Price equilibria under incomplete market coverage, under the parameter setting that $n = 1$, $\gamma = 0.9$, $q_H = 2q_L$, $\delta = 0.1$, $\bar{\theta} = 0.8$, and $g(q) = q$.

6. Conclusion

This paper studies a peer-to-peer platform that matches heterogeneous service providers with heterogenous consumers. Under a competitive search framework, we study how bilateral ratings, which reveal providers' service quality and consumers' serving costs, influence market competition and segmentation. The friction in our model comes not from search costs but from potential coordination failures: some providers may receive more than one applications although they can serve only one consumer, while other providers may receive no application although they can also serve a consumer. We find that two kinds of equilibria exist with bilateral ratings, although it is often the case that high-quality service providers would yield more net utility (i.e., net of price paid) for a consumer that gets served. In the first type of equilibria, high-quality service providers post a high price and serve all types of consumers, while low-quality service providers post a low price and serve only consumers with high serving costs (i.e., bad reviews). Bilateral ratings may improve the efficiency of matching in this case in the sense that good providers are more likely to be matched with good customers. In the second type of equilibria, both high- and low-quality service providers serve all consumers.

We find that across all the equilibria, consumers with high serving costs expect lower utility in equilibrium. In particular, a consumer may be willing to reimburse the service provider of her serving cost in order to be treated as a zero-cost consumer. In other words, consumers with high serving costs are subject to disproportionately low acceptance rates in equilibrium and pay an extremely high premium for their bad reputation.

Comparative statics suggest that equilibrium prices tend to be driven by the ratio of consumers to service providers and the fraction of high-quality providers in the market. When the ratio of consumers to providers is high, providers have more market power and prices get higher. When the fraction of high-quality providers is high, the competition of them becomes more fierce, which drives down their equilibrium prices, as well as the low-quality providers' prices. Interestingly, when the consumers' costs to serve increase across the market, it is possible for the prices to decrease. This is because the increased costs motivate the high-quality providers to compete on price more aggressively in order to get the low-cost consumer, causing prices to decrease across the board. Similarly, when the platform charges a higher commission rate so that the providers' margin decreases, we may also expect the equilibrium prices to decrease.

We also analyze the matching game with unilateral ratings, where only service providers are rated, and find that although bilateral ratings may generally lead to higher social welfare, it could soften service providers' competition through market segmentation and lead to higher equilibrium prices, compared with unilateral ratings.

Our analysis of the case of incomplete market coverage shows that when consumers have high serving costs in general and the fraction of high-quality service providers is high, competition among providers can drive down prices so much that the equilibrium price of high-quality providers becomes lower than that of low-quality providers. In this case, the low-quality providers do not compete for the low-cost consumers and will choose to serve the high-cost consumers at a premium price.

There are several directions for future research. First, we have assumed that customer reviews can accurately reveal the customer's type (i.e., her serving cost). In reality, there are strategic and fake reviews, which might make the ratings less informative. However, our model can still provide some useful insights as long as the overall review quality is informative to allow a significant amount of uncertainty about the cost to serve the customer to be resolved, which is probably true for repeatable service interactions over a long time horizon where reputation systems tend to be informative and truthful due to the risks of future punishments. With that said, it may be interesting to study the dynamic process of the review systems and other effects of reviews on peer-to-peer platforms (e.g., the revelation of other types of consumer information such as demographic information). Second, we only consider the matching game of a single round. It may be interesting to extend the current model to multiple rounds of matching and further explore the dynamics in the matching outcomes.

APPENDIX

Proof of Theorem 1:

Proof. Let us start with the following lemmas.

Lemma A1: *Suppose $q_L - p_L > 0$. If there exists $\theta' \in [0, \bar{\theta}]$ such that $x_H(\theta') > 0$ and $x_L(\theta') = 0$, then we have that $x_H(\theta) > 0$ and $x_L(\theta) = 0$ for $\forall \theta \in [0, \theta']$.*

Proof. The lemma is obviously true for $\theta' = 0$. For $\theta' > 0$, we prove the lemma by contradiction. Suppose there exists $\theta' \in (0, \bar{\theta}]$ such that $x_H(\theta') > 0$ and $x_L(\theta') = 0$, and there exists $\theta'' \in [0, \theta')$ such that $x_H(\theta'') = 0$ or $x_L(\theta'') > 0$. First, note that by equation (1) and $f(\cdot) > 0$, the condition that $x_H(\theta'') = 0$ or $x_L(\theta'') > 0$ is equivalent to $x_L(\theta'') > 0$.

According to consumers' application strategy in equation (3), the condition $x_H(\theta') > 0$ and $x_L(\theta') = 0$ imply that,

$$U(\theta') = e^{-\int_0^{\theta'} x_H(t)\mathrm{d}t}(q_H - p_H) > e^{-\int_0^{\theta'} x_L(t)\mathrm{d}t}(q_L - p_L),$$
$$\text{i.e., } e^{\int_0^{\theta'} (x_H(t) - x_L(t))\mathrm{d}t} < \frac{q_H - p_H}{q_L - p_L}. \tag{i}$$

Similarly, by applying equation (3) to the condition $x_L(\theta'') > 0$, we have that,

$$U(\theta'') = e^{-\int_0^{\theta''} x_L(t)\mathrm{d}t}(q_L - p_L) \geq e^{-\int_0^{\theta''} x_H(t)\mathrm{d}t}(q_H - p_H),$$
$$\text{i.e., } e^{\int_0^{\theta''} (x_H(t) - x_L(t))\mathrm{d}t} \geq \frac{q_H - p_H}{q_L - p_L}. \tag{ii}$$

Combining equations (i) and (ii), we have that,

$$e^{\int_0^{\theta''} (x_H(t) - x_L(t))\mathrm{d}t} \geq \frac{q_H - p_H}{q_L - p_L} > e^{\int_0^{\theta'} (x_H(t) - x_L(t))\mathrm{d}t}.$$
$$\text{i.e., } \int_{\theta''}^{\theta'} (x_H(t) - x_L(t))\,\mathrm{d}t < 0. \tag{iii}$$

Meanwhile, by the normalization condition in equation (1), we know that $x_H(\theta') - x_L(\theta') = \frac{n}{\gamma} f(\theta') > 0$. Because both $x_H(\theta)$ and $x_L(\theta)$ are piecewise continuous, we have that $x_H(\theta) - x_L(\theta)$ is piecewise continuous. Thus, there exists a neighborhood around θ', such that for all θ within the neighborhood, $x_H(\theta) - x_L(\theta) > \frac{n}{2\gamma} f(\theta') > 0$. Without loss of generality, let us assume that $x_H(\theta) - x_L(\theta)$ is left-continuous, so the neighborhood takes the form of $[\theta' - \varepsilon, \theta']$, where $\varepsilon > 0$. Similarly, if $x_H(\theta) - x_L(\theta)$ is right-continuous instead, the neighborhood takes the form of $[\theta', \theta' + \varepsilon]$, under which case, we can redefine θ' as $\theta' + \varepsilon$ for the following discussion. To summarize, we have argued that $x_H(\theta) - x_L(\theta) > \frac{n}{2\gamma} f(\theta') > 0$ for $\theta \in [\theta' - \varepsilon, \theta']$. Now we can rewrite the inequality (iii) as the following,

$$
\begin{aligned}
0 > \int_{\theta''}^{\theta'} (x_H(t) - x_L(t))\,\mathrm{d}t = \int_{\theta''}^{\theta'-\varepsilon} (x_H(t) - x_L(t))\,\mathrm{d}t + \int_{\theta'-\varepsilon}^{\theta'} (x_H(t) - x_L(t))\,\mathrm{d}t \\
> \int_{\theta''}^{\theta'-\varepsilon} (x_H(t) - x_L(t))\,\mathrm{d}t + \frac{n}{2\gamma} f(\theta')\varepsilon > 0,
\end{aligned}
$$

which is a contradiction. The last inequality above is due to the fact that inequality (iii) is valid for any θ' and θ'' that satisfy their definitions, so we can let θ'' and $\theta' - \varepsilon$ be infinitely close to each other, and consequently $\int_{\theta''}^{\theta'-\varepsilon} (x_H(t) - x_L(t))\,dt$ can be infinitely close to zero. Therefore, we have proved the original statement in Lemma A1. ■

By the exactly same logic, we can prove the following lemma, so its proof is omitted.

LEMMA A2: *Suppose $q_H - p_H > 0$. If there exists $\theta' \in \left[0, \bar{\theta}\right]$ such that $x_L(\theta') > 0$ and $x_H(\theta') = 0$, then we have that $x_L(\theta) > 0$ and $x_H(\theta) = 0$ for $\forall \theta \in [0, \theta']$.*

Now, let us prove Theorem 1. Let us first consider the case that $q_H - p_H \geq q_L - p_L > 0$. By the normalization condition in equation (1), we have that $0 \leq x_H(\theta) \leq \frac{n}{\gamma} f(\theta)$ and $0 \leq x_L(\theta) \leq \frac{n}{1-\gamma} f(\theta)$. This implies that,

$$
\begin{aligned}
b_H(\theta)(q_H - p_H) &= e^{-\int_0^\theta x_H(t)\mathrm{d}t}(q_H - p_H) \\
&\geq e^{-\int_0^\theta \frac{n}{\gamma} f(t)\mathrm{d}t}(q_H - p_H) \\
&= e^{-\frac{n}{\gamma}F(\theta)}(q_H - p_H) \\
&\geq e^{-\frac{n}{\gamma}F(\theta)} \frac{q_H - p_H}{q_L - p_L}(q_L - p_L) e^{-\int_0^\theta x_L(t)\mathrm{d}t} \\
&= e^{-\frac{n}{\gamma}F(\theta)} \frac{q_H - p_H}{q_L - p_L} \times b_L(\theta)(q_L - p_L) \\
&> b_L(\theta)(q_L - p_L), \text{ when } \theta < \theta_H.
\end{aligned}
$$

By equation (3), the above inequality implies that $x_H(\theta) = \frac{n}{\gamma} f(\theta)$ and $x_L(\theta) = 0$ for $\theta \in [0, \theta_H)$. If $\theta_H \geq \bar{\theta}$, we have effectively determined $x_j(\theta)$ ($j \in \{H, L\}$) for all $\theta \in \left[0, \bar{\theta}\right]$; on the other hand, if $\theta_H < \bar{\theta}$, we still need to determine $x_j(\theta)$ ($j \in \{H, L\}$) for $\theta \in \left[\theta_H, \bar{\theta}\right]$. Now consider the case that $\theta_H < \bar{\theta}$. Let us try to pin down $x_j(\theta)$ ($j \in \{H, L\}$) for $\theta \in \left(\theta_H, \bar{\theta}\right]$ first, and then we will determine $x_j(\theta_H)$ ($j \in \{H, L\}$).

First, we know that it is impossible for $x_L(\theta) > 0$ and $x_H(\theta) = 0$ for $\theta \in \left(\theta_H, \bar{\theta}\right]$, because by Lemma A2, this will imply $x_L(\theta) > 0$ and $x_H(\theta) = 0$ for $\theta \in [0, \theta_H)$, which is a contradiction.

Second, we also know that it is impossible for $x_H(\theta) > 0$ and $x_L(\theta) = 0$ for $\theta \in (\theta_H, \bar{\theta}]$, because by Lemma A1, this will imply that for $\theta \in (\theta_H, \bar{\theta}]$,

$$b_H(\theta)(q_H - p_H) = e^{-\int_0^\theta \frac{n}{\gamma} f(t)\mathrm{d}t}(q_H - p_H) = e^{-\frac{n}{\gamma}F(\theta)} \frac{q_H - p_H}{q_L - p_L}(q_L - p_L) < (q_L - p_L) = b_L(\theta)(q_L - p_L),$$

which is a contradiction. Therefore, we must have $x_H(\theta) > 0$ and $x_L(\theta) > 0$ for $\theta \in (\theta_H, \bar{\theta}]$. By equation (3), this implies that,

$$\begin{aligned} e^{-\int_0^\theta x_L(t)\mathrm{d}t}(q_L - p_L) &= e^{-\int_0^\theta x_H(t)\mathrm{d}t}(q_H - p_H) \\ \text{i.e., } e^{-\int_{\theta_H}^\theta x_L(t)\mathrm{d}t}(q_L - p_L) &= e^{-\int_0^{\theta_H} x_H(t)\mathrm{d}t} e^{-\int_{\theta_H}^\theta x_H(t)\mathrm{d}t}(q_H - p_H) \\ \text{i.e., } e^{-\int_{\theta_H}^\theta x_L(t)\mathrm{d}t} &= e^{-\int_{\theta_H}^\theta x_H(t)\mathrm{d}t} \\ \text{i.e., } \int_{\theta_H}^\theta (x_H(t) - x_L(t))\,\mathrm{d}t &= 0. \end{aligned}$$

Notice that the above equality is valid for $\forall\theta \in (\theta_H, \bar{\theta}]$, we must have $x_H(\theta) = x_L(\theta)$ for $\forall\theta \in (\theta_H, \bar{\theta}]$. By piece-wise continuity of $x_H(\theta)$ and $x_L(\theta)$, we must also have that $x_H(\theta_H) = x_L(\theta_H)$. By the normalization condition in equation (1), we have $x_H(\theta) = x_L(\theta) = nf(\theta)$ for $\forall\theta \in [\theta_H, \bar{\theta}]$.

We have completely proved the theorem for the case $q_H - p_H \geq q_L - p_L > 0$ above. The proof for the other case with $q_L - p_L \geq q_H - p_H > 0$ follows the exactly same line, and thus is omitted. Lastly, $U(\theta)$ can be calculated by equation (3) given $x_H(\theta)$ and $x_L(\theta)$. ■

Proof of The Concavity Condition in Equation (10):

Proof. For $p_j^0 \in [q_j - U(0), q_j]$, $\pi_j^0(p_j^0; p_H, p_L)$ is given by the second case in equation (5). $\pi_j^0(p_j^0; p_H, p_L)$ is concave if and only if,

$$\frac{\partial \pi_j^0(p_j^0; p_H, p_L)}{\partial (p_j^0)^2} = -\frac{1}{(q_j - p_j^0)^3}\Bigg\{ 2\left[(1-\delta)q_j - \bar{\theta}g(g_j)\right] U(\bar{\theta}) + g(q_j)\left[2\int_{U^{-1}(q_j - p_j^0)}^{\bar{\theta}} U(\theta)\mathrm{d}\theta + \frac{(q_j - p_j^0)^2}{U'\left(U^{-1}(q_j - p_j^0)\right)}\right]\Bigg\} \leq 0. \quad \text{(iv)}$$

By equation (3), we know that $U(\theta) = (q_{j^*} - p_{j^*})e^{-\int_0^\theta x_{j^*}(t)\mathrm{d}t}$, where $j^* \in \{H, L\}$ is defined by $x_{j^*} > 0$. Therefore, $U'(\theta) = -U(\theta)x_{j^*}(\theta)$. By substituting this equality back to inequality (iv), we can rearrange and rewrite inequality (iv) as the following inequality,

$$x_{j^*}\left(U^{-1}(q_j-p_j^0)\right) \geq \frac{(q_j-p_j^0)}{2\left[\left(\frac{(1-\delta)q_j}{g(q_j)}-\bar{\theta}\right)U(\bar{\theta})+\int_{U^{-1}(q_j-p_j^0)}^{\bar{\theta}}U(t)\mathrm{d}t\right]}.$$

Denote $\theta = U^{-1}(q_j - p_j^0)$, which ranges in $[0, \bar{\theta}]$. Notice that x_{j^*} is given by Theorem 1. We can rewrite the above inequality as equation (10). ■

Proof of Theorem 2:

Proof. To prove the existence of Nash equilibria with $q_H - p_H^* = q_L - p_L^* = \varepsilon \in (0, \bar{\varepsilon}]$, we need to show that condition (8) is satisfied for $\varepsilon \in (0, \bar{\varepsilon}]$. In fact, condition (8) can be simplified as,

$$\begin{cases} q_H - p_H^* = q_L - p_L^* = \varepsilon, \\ (1-\delta)\varepsilon - \dfrac{1}{n}\bar{\theta}g(q_j) - e^{-n}\left[(1-\delta)q_j - \dfrac{n+1}{n}\bar{\theta}g(q_j)\right] \leq 0, \ j \in \{H, L\}. \end{cases}$$

This implies that $\varepsilon \leq \bar{\varepsilon}$. To show that $\bar{\varepsilon} \geq \frac{\bar{\theta}g(g_L)}{2(1-\delta)n} > 0$, we only need to take $\theta = 0$ and $q_H - p_H^* = q_L - p_L^*$ in concavity condition (10).

To show that in equilibrium, $q_H - p_H^* \geq q_L - p_L^*$, we need to show that equation (7) never happens in equilibrium, given the concavity condition (10). Equivalently, we need to show that under the concavity condition (10),

$$\text{If} \begin{cases} q_H - p_H^* < q_L - p_L^* \\ (1-\delta)(q_H - p_H^*)^2 - \left[(1-\delta)q_H - g(q_H)\bar{\theta}\right]U(\bar{\theta}) - g(q_H)\displaystyle\int_{\theta_L}^{\bar{\theta}} U(\theta)\mathrm{d}\theta = 0 \end{cases},$$

$$\text{then } (1-\delta)(q_L - p_L^*)^2 - \left[(1-\delta)q_L - g(q_L)\bar{\theta}\right]U(\bar{\theta}) - g(q_L)\int_0^{\bar{\theta}} U(\theta)\mathrm{d}\theta > 0.$$

By substituting the expression of $U(\theta)$ in the second case of Theorem 1, the above statement can be equivalently written as,

$$\text{If} \begin{cases} q_H - p_H^* < q_L - p_L^* \\ (1-\delta)(q_H - p_H^*) - \dfrac{1}{n}g(q_H)\bar{\theta} - \left[(1-\delta)q_H - \dfrac{n+1}{n}g(q_H)\bar{\theta}\right]e^{-n}\left(\dfrac{q_H - p_H^*}{q_L - p_L^*}\right)^{-(1-\gamma)} = 0 \end{cases},$$

$$\text{then } (1-\delta)(q_L - p_L^*) - \frac{1}{n}g(q_L)\bar{\theta}\left[(1-\gamma) + \gamma\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)\right]$$
$$-\left[(1-\delta)q_L - \left(1+\frac{1}{n}\right)g(q_L)\bar{\theta}\right]e^{-n}\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma} > 0.$$

In fact, we have that,

$$
\begin{aligned}
&(1-\delta)(q_L - p_L^*) - \frac{1}{n} g(q_L)\bar{\theta}\left[(1-\gamma) + \gamma\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)\right] \\
&- \left[(1-\delta)q_L - \left(1+\frac{1}{n}\right) g(q_L)\bar{\theta}\right] e^{-n}\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma} \\
=& \left(\frac{q_L - p_L^*}{q_H - p_H^*}\right)\Bigg\{(1-\delta)(q_H - p_H^*) - \frac{1}{n} g(q_L)\bar{\theta}\left[(1-\gamma) + \gamma\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)\right]\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) \\
&+ \frac{1}{2n} g(q_L)\bar{\theta} e^{-n}\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1} - \left[(1-\delta)q_L - \left(1+\frac{1}{2n}\right) g(q_L)\bar{\theta}\right] e^{-n}\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1}\Bigg\} \\
\geq& \left(\frac{q_L - p_L^*}{q_H - p_H^*}\right)\Bigg\{(1-\delta)(q_H - p_H^*) - \frac{1}{n} g(q_L)\bar{\theta}\left[(1-\gamma) + \gamma\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)\right]\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) \\
&+ \frac{1}{2n} g(q_L)\bar{\theta} e^{-n}\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1} - \left[(1-\delta)q_H - \left(1+\frac{1}{2n}\right) g(q_H)\bar{\theta}\right] e^{-n}\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1}\Bigg\} \qquad \text{(v)} \\
=& \left(\frac{q_L - p_L^*}{q_H - p_H^*}\right)\Bigg\{(1-\delta)(q_H - p_H^*) - \frac{1}{n} g(q_L)\bar{\theta}\left[(1-\gamma) + \gamma\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)\right]\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) \\
&+ \frac{1}{2n} g(q_L)\bar{\theta} e^{-n}\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1} - \left[(1-\delta)q_H - \left(1+\frac{1}{2n}\right) g(q_H)\bar{\theta}\right] e^{-n}\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1} \\
&- \left[(1-\delta)(q_H - p_H^*) - \frac{1}{n} g(q_H)\bar{\theta} - \left[(1-\delta)q_H - \left(1+\frac{1}{n}\right) g(q_H)\bar{\theta}\right] e^{-n}\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{-(1-\gamma)}\right]\Bigg\} \\
=& \left(\frac{q_L - p_L^*}{q_H - p_H^*}\right)\Bigg\{\frac{1}{n} g(q_H)\bar{\theta}\left[1 - \frac{1}{2} e^{-n}\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1}\right] \\
&- \frac{1}{n} g(q_L)\bar{\theta}\left[(1-\gamma) + \gamma\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) - \frac{1}{2} e^{-n}\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma}\right]\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) \\
&+ \left[(1-\delta)q_H - \left(1+\frac{1}{n}\right) g(q_H)\bar{\theta}\right] e^{-n}\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{-(1-\gamma)}\left[1 - \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{2}\right]\Bigg\} \\
\geq& \left(\frac{q_L - p_L^*}{q_H - p_H^*}\right)\Bigg\{\frac{1}{n} g(q_H)\bar{\theta}\left[1 - \frac{1}{2} e^{-n}\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1}\right] \\
&- \frac{1}{n} g(q_L)\bar{\theta}\left[(1-\gamma) + \gamma\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) - \frac{1}{2} e^{-n}\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma}\right]\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) \\
&- \frac{1}{2n} g(q_H)\bar{\theta}\left[1 - \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{2}\right]\Bigg\}. \qquad \text{(vi)}
\end{aligned}
$$

Inequality (v) is due to that

$$(1-\delta)q_L - \left(1+\frac{1}{2n}\right) g(q_L)\bar{\theta} \le (1-\delta)q_H - \left(1+\frac{1}{2n}\right) g(q_H)\bar{\theta}.$$

This inequality is obviously true if $g(q) = 1$. To show it is true if $g(q) = q$, we need to show that $(1-\delta) - \left(1+\frac{1}{2n}\right)\bar{\theta} \ge 0$. In fact, letting $\theta = \bar{\theta}$ in equation (10), we have that,

$$\frac{n}{\bar{\theta}} \ge \frac{1}{2\left(\frac{(1-\delta)q_j}{g(q_j)} - \bar{\theta}\right)},$$

which implies that $(1-\delta) - \left(1+\frac{1}{2n}\right)\bar{\theta} \ge 0$ when $g(q) = q$.

Inequality (vi) is due to that

$$\left[(1-\delta)q_H - \left(1+\frac{1}{n}\right) g(q_H)\bar{\theta}\right] e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{-(1-\gamma)} \ge -\frac{1}{2n} g(q_H)\bar{\theta}.$$

This inequality can be obtained if letting $\theta = \theta_L$ and $j = H$ in equation (10).

To continue the derivation following equation (vi), we have,

$$\begin{aligned}
&\left(\frac{q_L - p_L^*}{q_H - p_H^*}\right) \left\{ \frac{1}{n} g(q_H)\bar{\theta} \left[1 - \frac{1}{2} e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1}\right] \right. \\
&- \frac{1}{n} g(q_L)\bar{\theta} \left[(1-\gamma) + \gamma\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) - \frac{1}{2} e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma}\right] \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) \\
&\left. - \frac{1}{2n} g(q_H)\bar{\theta} \left[1 - \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^2\right] \right\} \\
=& \left(\frac{q_L - p_L^*}{q_H - p_H^*}\right) \left\{ \frac{1}{n} g(q_H)\bar{\theta} \left[\frac{1}{2} + \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^2 - \frac{1}{2} e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1}\right] \right. \\
&\left. - \frac{1}{n} g(q_L)\bar{\theta} \left[(1-\gamma) + \gamma\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) - \frac{1}{2} e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma}\right] \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) \right\} \\
\ge& \left(\frac{q_L - p_L^*}{q_H - p_H^*}\right) \left\{ \frac{1}{n} g(q_L)\bar{\theta} \left[\frac{1}{2} + \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^2 - \frac{1}{2} e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma+1}\right] \right. \\
&\left. - \frac{1}{n} g(q_L)\bar{\theta} \left[(1-\gamma) + \gamma\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) - \frac{1}{2} e^{-n} \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)^{\gamma}\right] \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) \right\} \qquad \text{(vii)} \\
=& \left(\frac{q_L - p_L^*}{q_H - p_H^*}\right) \frac{1}{2n} g(q_L)\bar{\theta} \left[1 - \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)\right] \left[1 - \left(\frac{q_H - p_H^*}{q_L - p_L^*}\right) + 2\gamma\left(\frac{q_H - p_H^*}{q_L - p_L^*}\right)\right] > 0. \qquad \text{(viii)}
\end{aligned}$$

Inequality (vii) is due to $g(q_H) \ge g(q_L)$, and inequality (viii) is due to $0 < \frac{q_H - p_H^*}{q_L - p_L^*} < 1$. ■

Proof of Concavity of $\Pi_j^0(p_j^0; p_H, p_L)$:

Proof.

$$\frac{\partial^2 \Pi_j^0}{\partial (p_j^0)^2} = -\frac{W\left(-\frac{q_j-p_j^0}{U}e^{-\frac{q_j-p_j^0}{U}}\right)\left[\frac{q_j-p_j^0}{U}+W\left(-\frac{q_j-p_j^0}{U}e^{-\frac{q_j-p_j^0}{U}}\right)\right]}{2(q_j-p_j^0)^3\left[1+W\left(-\frac{q_j-p_j^0}{U}e^{-\frac{q_j-p_j^0}{U}}\right)\right]^3}$$
$$\times\Bigg\{-4(1-\delta)U(q_j-p_j^0)\left[1+W\left(-\frac{q_j-p_j^0}{U}e^{-\frac{q_j-p_j^0}{U}}\right)\right]^2$$
$$-2U\left[(1-\delta)p_j^0-\frac{\bar{\theta}}{2}g(q_j)\right]\left[\frac{q_j-p_j^0}{U}+3W\left(-\frac{q_j-p_j^0}{U}e^{-\frac{q_j-p_j^0}{U}}\right)+2W\left(-\frac{q_j-p_j^0}{U}e^{-\frac{q_j-p_j^0}{U}}\right)^2\right]\Bigg\},$$

where $W(z)$ is the product logarithm function, which is defined as the upper branch of the inverse function of $z = We^W$. We notice that $\frac{q_j-p_j^0}{U} \geq 1$, so $0 > W\left(-\frac{q_j-p_j^0}{U}e^{-\frac{q_j-p_j^0}{U}}\right) \geq -1 \geq -\frac{q_j-p_j^0}{U}$. We can further show that $\frac{q_j-p_j^0}{U}+3W\left(-\frac{q_j-p_j^0}{U}e^{-\frac{q_j-p_j^0}{U}}\right)+2W\left(-\frac{q_j-p_j^0}{U}e^{-\frac{q_j-p_j^0}{U}}\right)^2 \geq 0$. We also notice that $(1-\delta)p_j^0 - \frac{\bar{\theta}}{2}g(q_j) \geq 0$. According to all these inequalities, it is straightforward to verify that $\frac{\partial^2\Pi_j^0}{\partial(p_j^0)^2} \leq 0$, and thus $\Pi_j^0(p_j^0; p_H, p_L)$ is a concave function in p_j^0. ■

Proof of Theorem 3:

Proof. Because $\Pi_j^0(p_j^0; p_H, p_L)$ is continuous in p_j^0, p_H and p_L, and concave in p_j^0, existence of a pure strategy Nash equilibrium is guaranteed by classic results, such as Proposition 8.D.3 on page 260 of Mas-Collell et al. (1995), except that now we have symmetric infinite games. Cheng et al. (2004) extend the classic results to consider symmetric infinite games, and show that a symmetric pure-strategy Nash equilibrium exists with compact, convex strategy spaces and continuous, quasiconcave utility functions.

We prove $q_H - p_H^{**} > q_L - p_L^{**}$ by contradiction. Suppose $q_H - p_H^{**} \leq q_L - p_L^{**}$. We know that $(1-\delta)q_H - \frac{\bar{\theta}}{2}g(q_H) > (1-\delta)q_L - \frac{\bar{\theta}}{2}g(q_L)$ in the case that $q_H > q_L$, and $g(q) = 1$ or $g(q) = q$. Therefore, we have,

$$\frac{(1-\delta)(q_H-p_H)}{(1-\delta)q_H-\frac{\bar{\theta}}{2}g(q_H)} < \frac{(1-\delta)(q_L-p_L)}{(1-\delta)q_L-\frac{\bar{\theta}}{2}g(q_L)} < 1,$$

which implies that

$$\frac{(1-\delta)(q_H-p_H)}{(1-\delta)q_H-\frac{\bar{\theta}}{2}g(q_H)} - \ln\left[\frac{(1-\delta)(q_H-p_H)}{(1-\delta)q_H-\frac{\bar{\theta}}{2}g(q_H)}\right] > \frac{(1-\delta)(q_L-p_L)}{(1-\delta)q_L-\frac{\bar{\theta}}{2}g(q_L)} - \ln\left[\frac{(1-\delta)(q_L-p_L)}{(1-\delta)q_L-\frac{\bar{\theta}}{2}g(q_L)}\right].$$

Meanwhile, we know that,

$$1 \leq \frac{q_H - p_H}{U} \leq \frac{q_L - p_L}{U},$$

which implies that,

$$\frac{q_H - p_H}{U} - \ln\left(\frac{q_H - p_H}{U}\right) \leq \frac{q_L - p_L}{U} - \ln\left(\frac{q_L - p_L}{U}\right).$$

By equation (16), we know that

$$\frac{(1-\delta)(q_H - p_H)}{(1-\delta)q_H - \frac{\bar{\theta}}{2}g(q_H)} - \ln\left[\frac{(1-\delta)(q_H - p_H)}{(1-\delta)q_H - \frac{\bar{\theta}}{2}g(q_H)}\right] = \frac{q_H - p_H}{U} - \ln\left(\frac{q_H - p_H}{U}\right),$$

which implies that

$$\frac{(1-\delta)(q_L - p_L)}{(1-\delta)q_L - \frac{\bar{\theta}}{2}g(q_L)} - \ln\left[\frac{(1-\delta)(q_L - p_L)}{(1-\delta)q_L - \frac{\bar{\theta}}{2}g(q_L)}\right] < \frac{q_L - p_L}{U} - \ln\left(\frac{q_L - p_L}{U}\right).$$

This is a contradiction to equation (16). Therefore, we have that $q_H - p_H^{**} > q_L - p_L^{**}$. ■

Provider's Pricing Problem under Incomplete Market Coverage:

Let us first consider a service provider of type j posting price p_j^0. Similar to equation (5) in the case of complete market coverage, a low-type service provider's profit function can be written as

$$\pi_L^0(p_L^0; p_H, p_L) = \begin{cases} -(1-\delta)\left[U(0) - U(\bar{\theta})\right] \\ \quad -\frac{\int_0^{\bar{\theta}} \left[(1-\delta)q_L - g(q_L)\theta\right] U'(\theta) d\theta}{q_L - p_L^0}, & p_L^0 \leq q_L - U(0), \\ (1-\delta)U(\bar{\theta}) + (1-\delta)p_L^0 - g(q_L)U^{-1}(q_L - p_L^0) \\ \quad -\frac{(1-\delta)U(\bar{\theta})q_L + g(q_L)\left[\int_{U^{-1}(q_L - p_L^0)}^{\bar{\theta}} U(\theta)\mathrm{d}\theta - \bar{\theta}U(\bar{\theta})\right]}{q_L - p_L^0} & \text{otherwise.} \end{cases} \quad \text{(ix)}$$

Different from the case of complete market coverage, $U(\cdot)$ is no longer a continuous function when $\frac{q_H - p_H}{q_L - p_L} \geq e^{\frac{n}{\gamma}F\left(\frac{(1-\delta)p_H}{g(q_H)}\right)}$. Therefore, $U^{-1}(\cdot)$ is not well defined. By redefining $U^{-1}(u) = \inf\left\{x \in [0, \bar{\theta}] \middle| U(x) \leq u\right\}$, one can show that equation (ix) still holds.

Consider now a high-type service provider, whose profit function is a little different from equation (5) in that the service provider would not serve any consumer with cost type $\theta \in ((1-\delta)p_H/g(q_H), \bar{\theta}]$ and therefore the upper limit of the integral in equation (5) is replaced by $(1-\delta)p_H/g(q_H)$ in writing down the service provider's profit function:

$$\pi_H^0(p_H^0; p_H, p_L) = \begin{cases} -(1-\delta)\left[U(0) - U\left(\frac{(1-\delta)p_H^0}{q_H}\right)\right] \\ \quad -\frac{\int_0^{\frac{(1-\delta)p_H^0}{q_H}} \left[(1-\delta)q_H - g(q_H)\theta\right] U'(\theta)d\theta}{q_H - p_H^0}, & p_H^0 \leq q_H - U(0), \\ (1-\delta)U\left(\frac{(1-\delta)p_H^0}{q_H}\right) + (1-\delta)p_H^0 \\ \quad -g(q_H)U^{-1}(q_H - p_H^0) - \frac{(1-\delta)U\left(\frac{(1-\delta)p_H^0}{q_H}\right)q_H}{q_H - p_H^0} \\ \quad -\frac{g(q_H)\left[\int_{U^{-1}(q_H - p_H^0)}^{\frac{(1-\delta)p_H^0}{q_H}} U(\theta)\mathrm{d}\theta - \frac{(1-\delta)p_H^0}{q_H}U\left(\frac{(1-\delta)p_H^0}{q_H}\right)\right]}{q_H - p_H^0}, & \text{otherwise.} \end{cases} \tag{x}$$

Following similar analysis in Section 2.2, we can write down the first-order optimality conditions to the profit maximization problems whose objectives are given in equations (ix) and (x), along with the incomplete market coverage condition, as follows:

$$\begin{cases} p_H^* < \bar{\theta}g(q_H)/(1-\delta) \\ p_L^* \geq \bar{\theta}g(q_L)/(1-\delta) \\ (1-\delta)(q_H - p_H^*)^2 - (1-\delta)(q_H - p_H^*)U\left(\frac{(1-\delta)p_H^*}{q_H}\right) - g(q_H)\int_0^{\frac{(1-\delta)p_H^*}{q_H}} U(\theta)\mathrm{d}\theta \leq 0 \\ (1-\delta)(q_L - p_L^*)^2 - \left[(1-\delta)q_L - g(q_L)\bar{\theta}\right]U(\bar{\theta}) - g(q_L)\int_{\min\left\{\theta_H, \frac{(1-\delta)p_H^*}{q_H}\right\}}^{\bar{\theta}} U(\theta)\mathrm{d}\theta = 0 \end{cases} \tag{xi}$$

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